IEEE INFOCOM 2021
Virtual Conference

A Worst-Case Approximate Analysis of Peak Age-of-Information Via Robust Queueing Approach

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Joint work with

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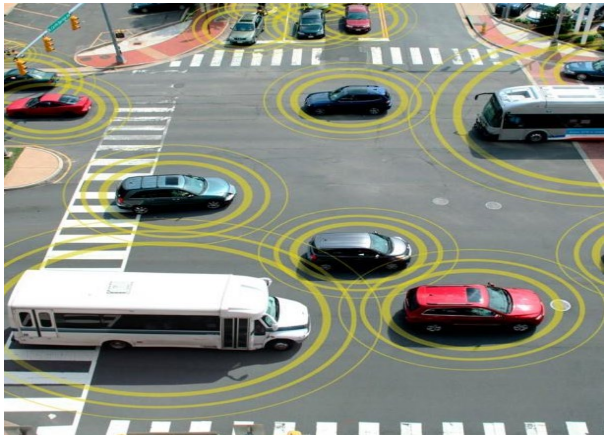
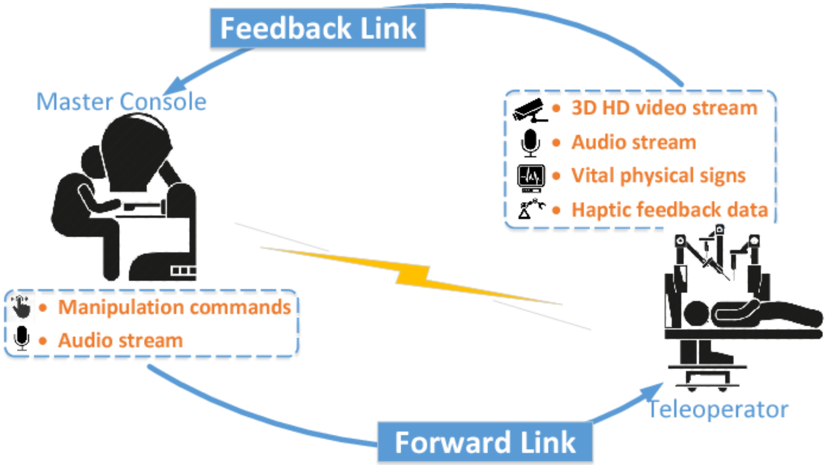
Freshness Matters

- Real-time services are ubiquitous
  - Intelligent transportation systems & vehicular networks
  - Sensor networks (for environmental/health monitoring)
  - Wireless channel feedback, news feeds, weather updates, etc.



Intelligent vehicular networks



Sensor networks

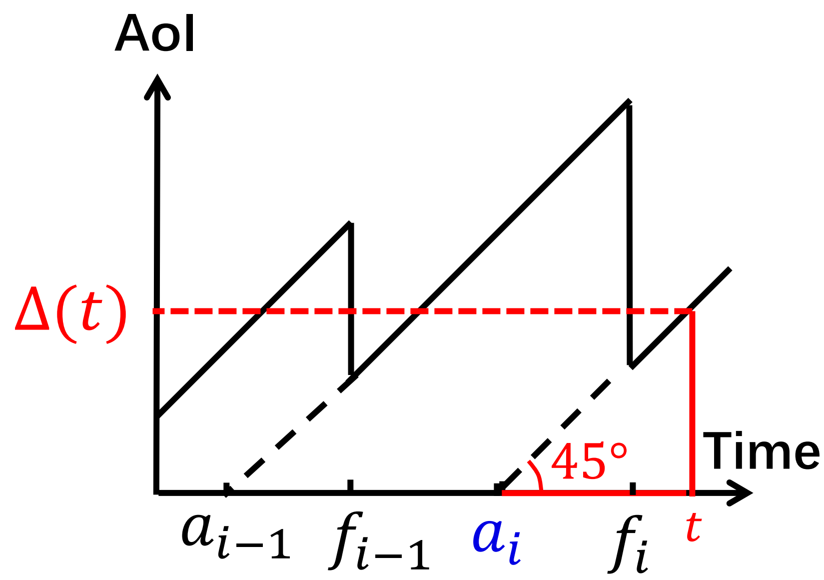
https://www.networkworld.com/article/2881654/security0/wireless-cyber-security-in-your-car-stinks.html

Metric of Freshness

- **Age-of-information (AoI)**: The time difference between current time and the generation time of the latest received update

If update i is generated at a_i and delivered at f_i, then AoI at time t is

$$\Delta(t) = t - \max\{a_i : f_i \le t\}$$



Sanjit Kaul, Roy Yates, and Marco Gruteser. "Real-time status: How often should one update?" Proceedings of IEEE INFOCOM 2012.

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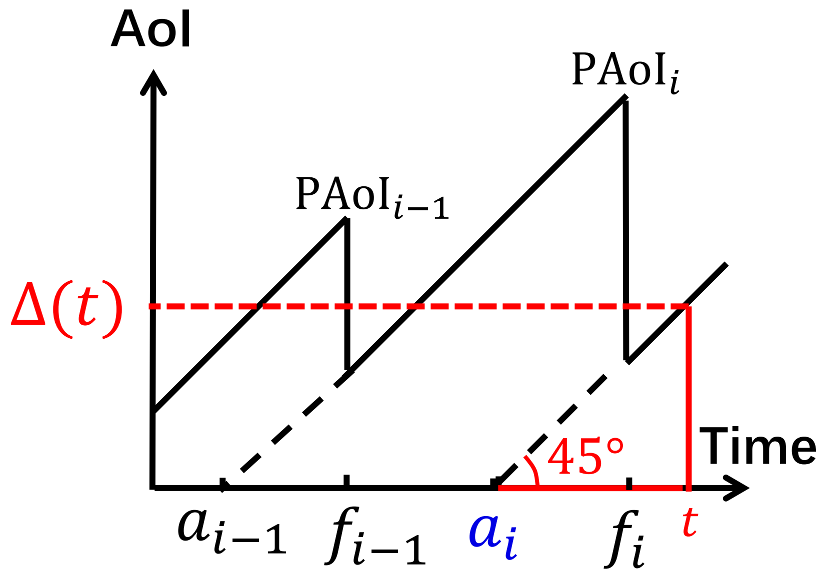
- **Peak Age-of-Information (PAoI)**: The maximum value of the AoI before it drops



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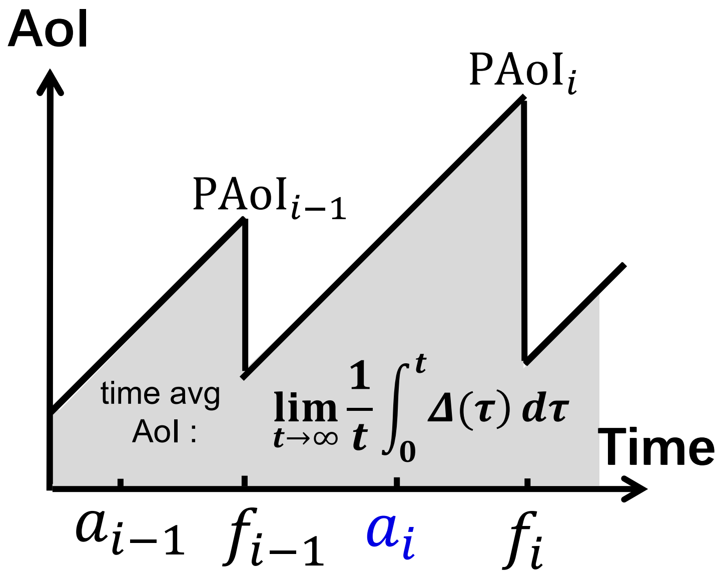

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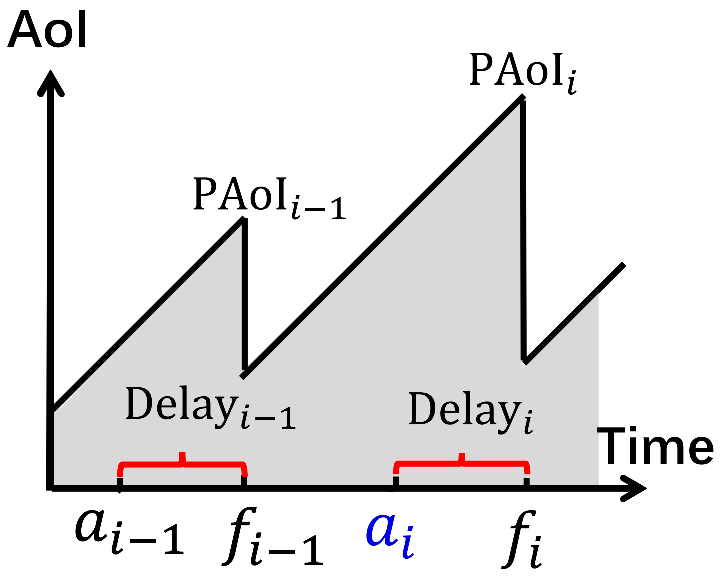
- **Delay**: The time difference between the generation time and the delivery time



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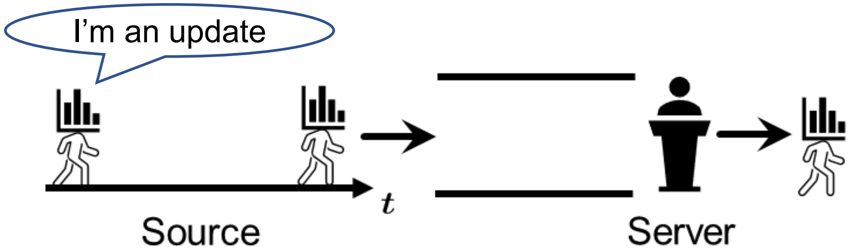
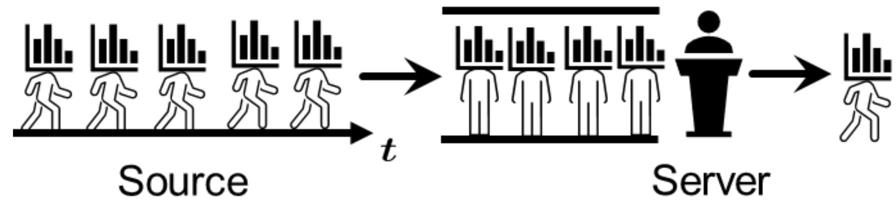
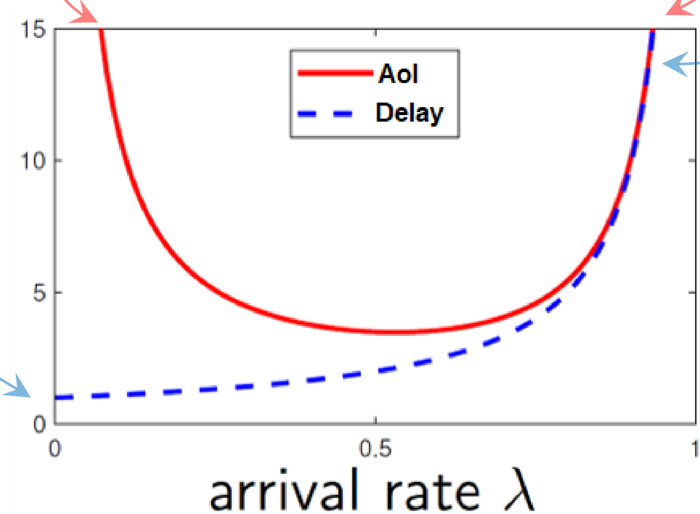
AoI vs. Delay

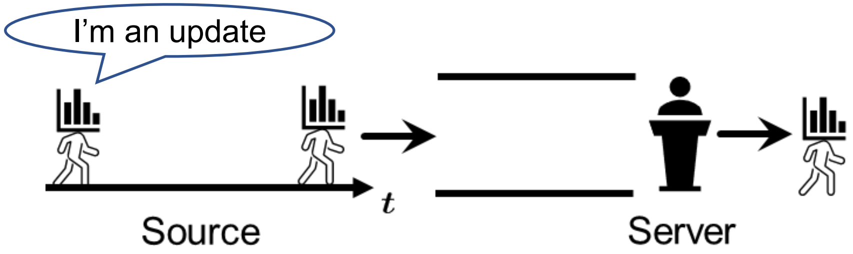


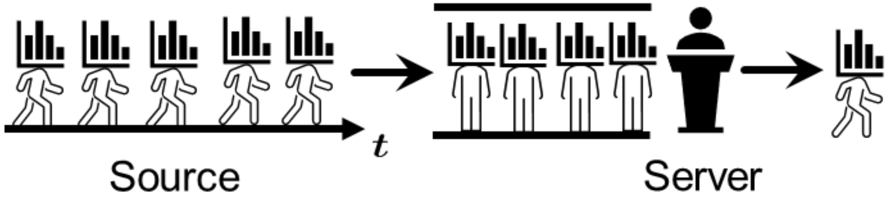


- Low arrival rate
  - Empty buffer → low delay
  - Infrequent updates → long interarrival time & high AoI
- Large arrival rate
  - Full buffer → high delay
  - Become stale while waiting→ high AoI



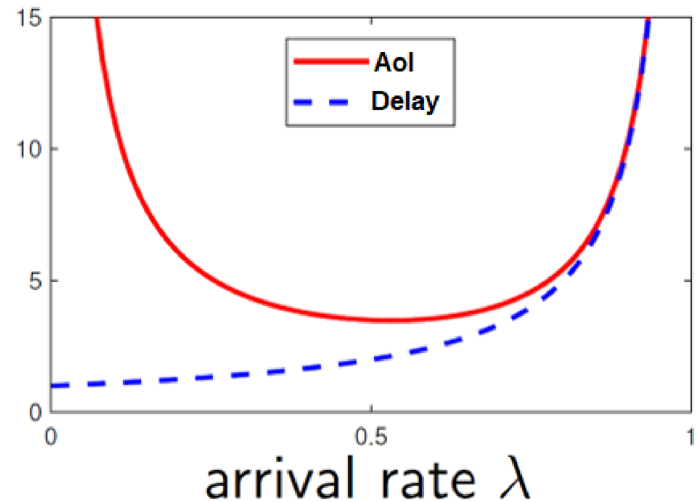
AoI vs. Delay





AoI/PAoI are large under both

- **Low-load regime**
- **High-load regime**



Related Work

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- AoI studies:
  - Assume **certain distributions** for arrival and service processes:
    - AoI in M/M/1,M/D/1, D/M/1 queues under the FCFS policy [Kaul et al. ‘12]
    - AoI in M/Gamma/1 queue under the PLCFS policy [Najm et al. ‘16]
  - Assume ***i.i.d.* arrival and service process**:
    - AoI in GI/GI/1, M/GI/1, and GI/M/1 queues [Inoue et al. ‘19]
- Delay studies:
  - Assume ***i.i.d.* arrival and service processes**:
    - Kingman’s bound [Kingman et al. ‘70, Ciucu et al. ‘18]
  - Only focus on **high-load regime**:
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Our contributions:

- Applying the robust queueing theory to analyzing PAoI performance

Related Work

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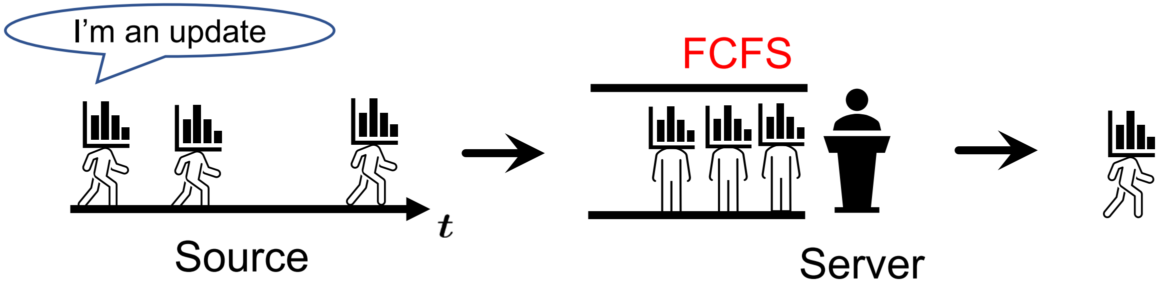
Our contributions:

- Applying the robust queueing theory to analyzing PAoI performance
- Approximating expected PAoI well under both high and low load regimes

Single-Source System (G/G/1)



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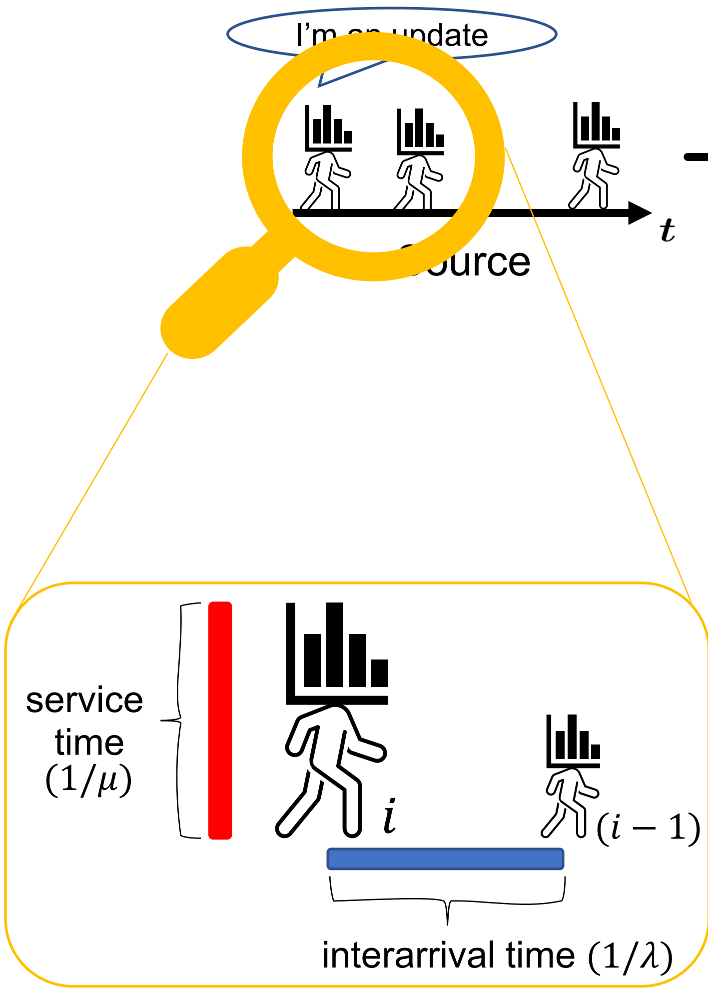

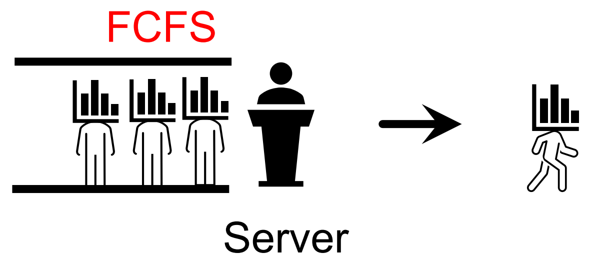

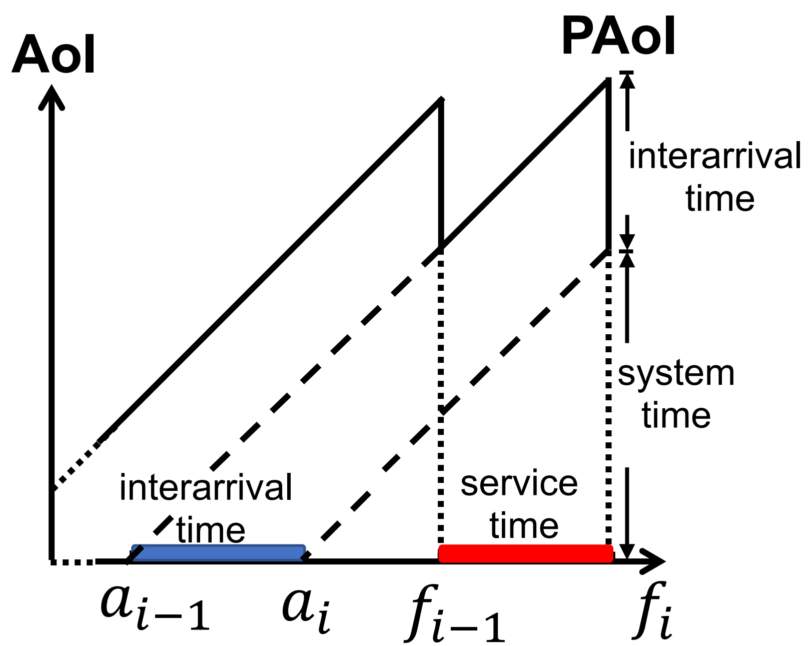

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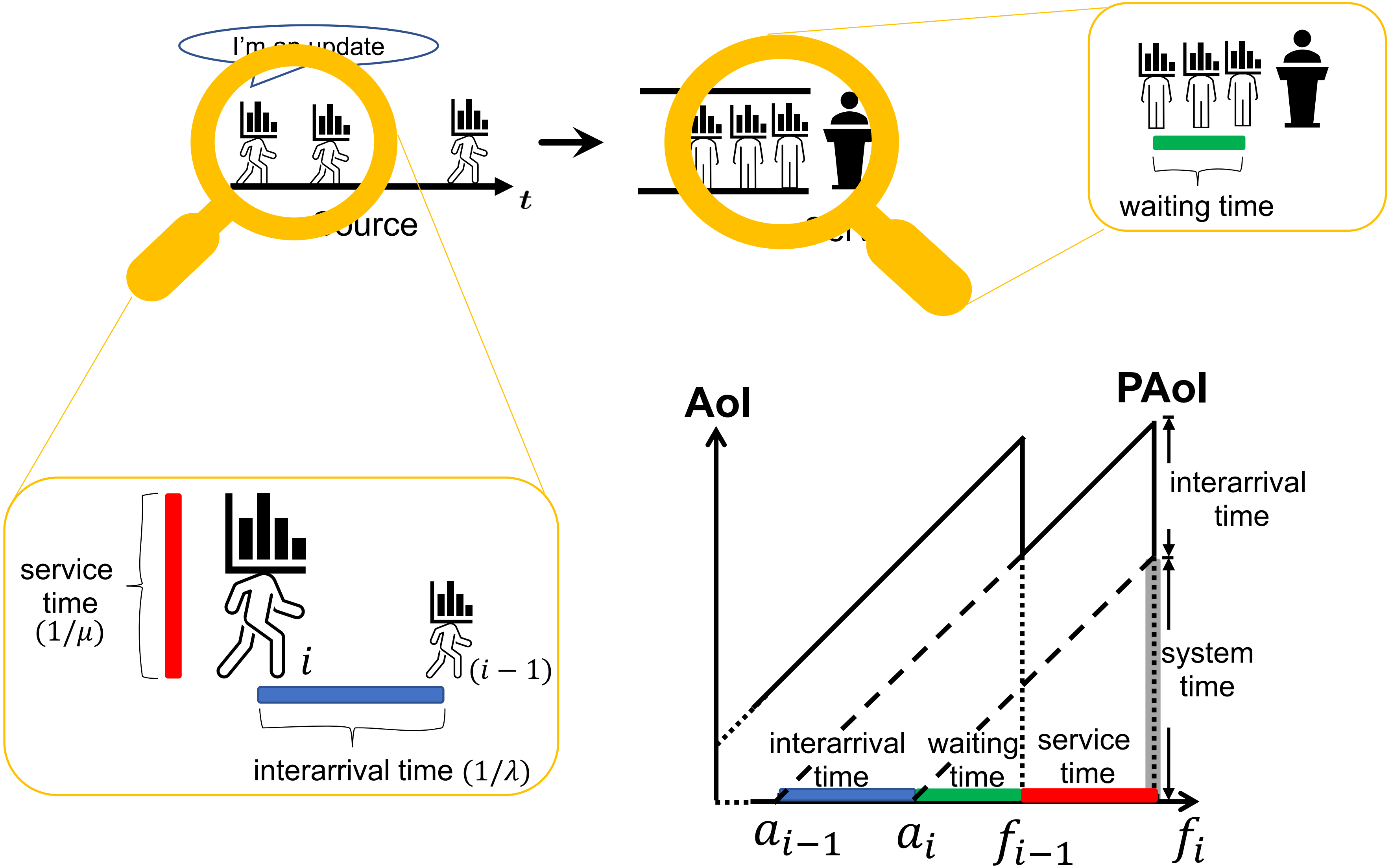
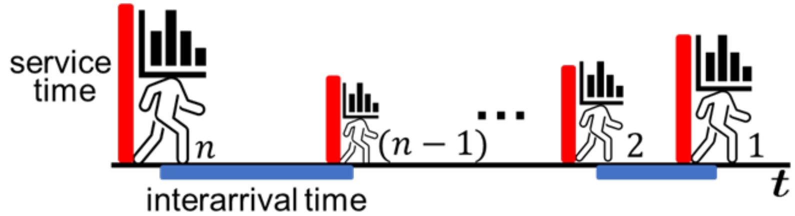


The Robust-Queueing Approach

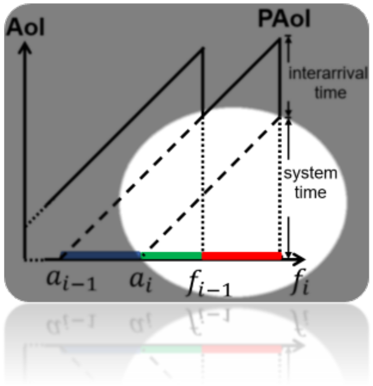
Uncertainty sets of arrival/service process



- Modeling their stochastic properties

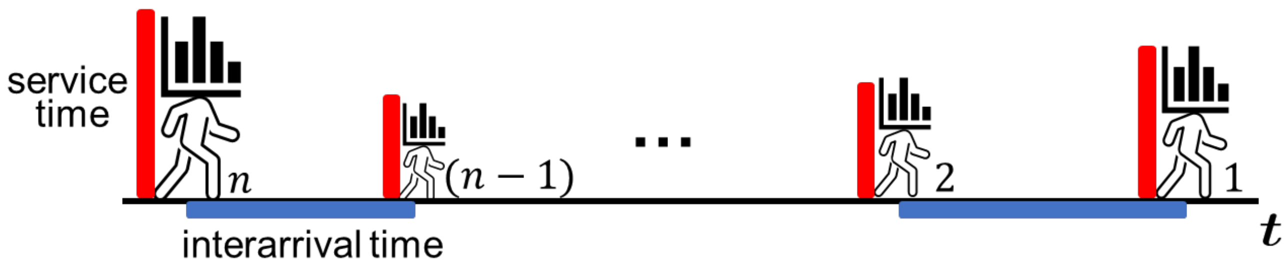
- Getting rid of **unrealistic assumptions** (e.g., memoryless properties or *i.i.d.*)

Worst-case analysis of system time



- Used to approximate the expected PAoI
- Work well under **both low-load and high-load regimes**

Motivation of Uncertainty Set



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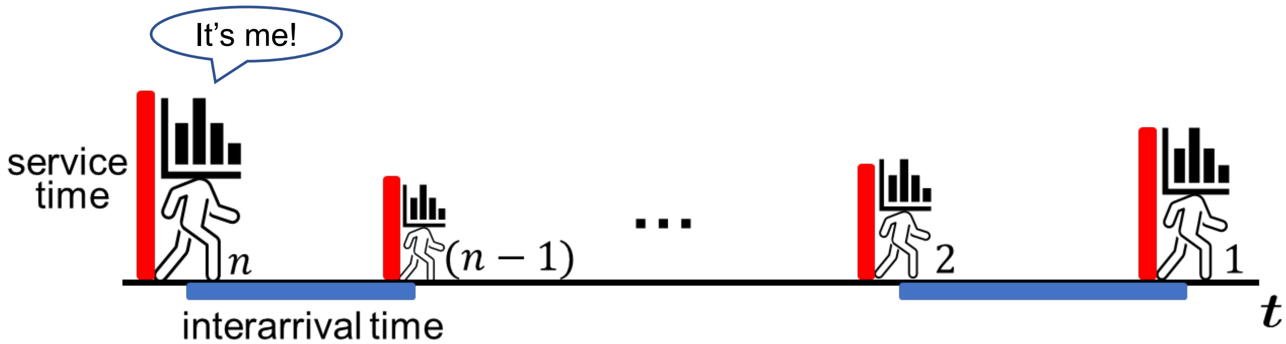

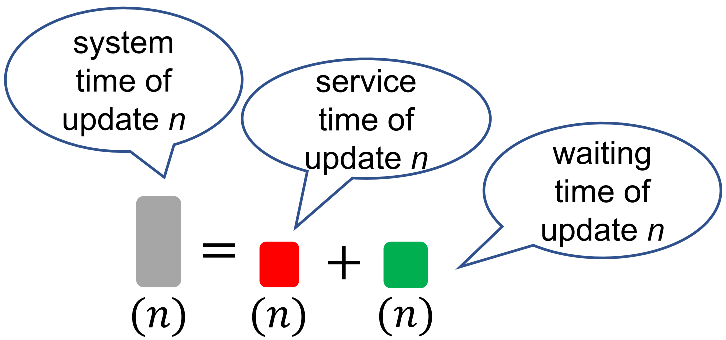

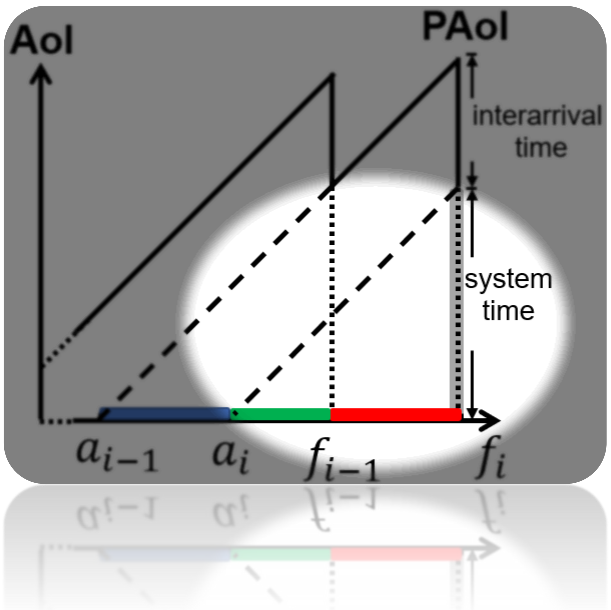

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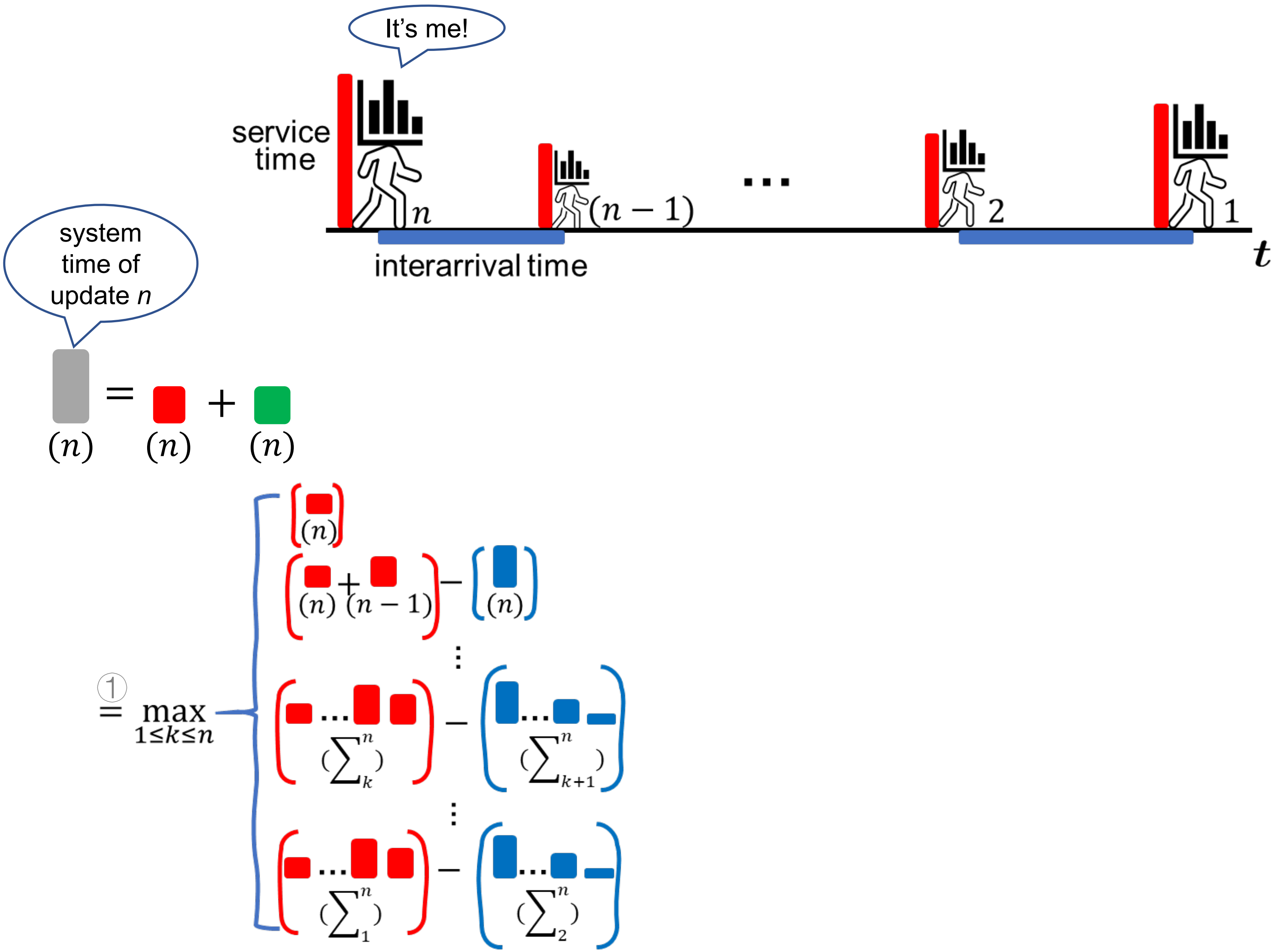

① D. V. Lindley, "The theory of queues with a single server," in Mathematical Proceedings of the Cambridge Philosophical Society

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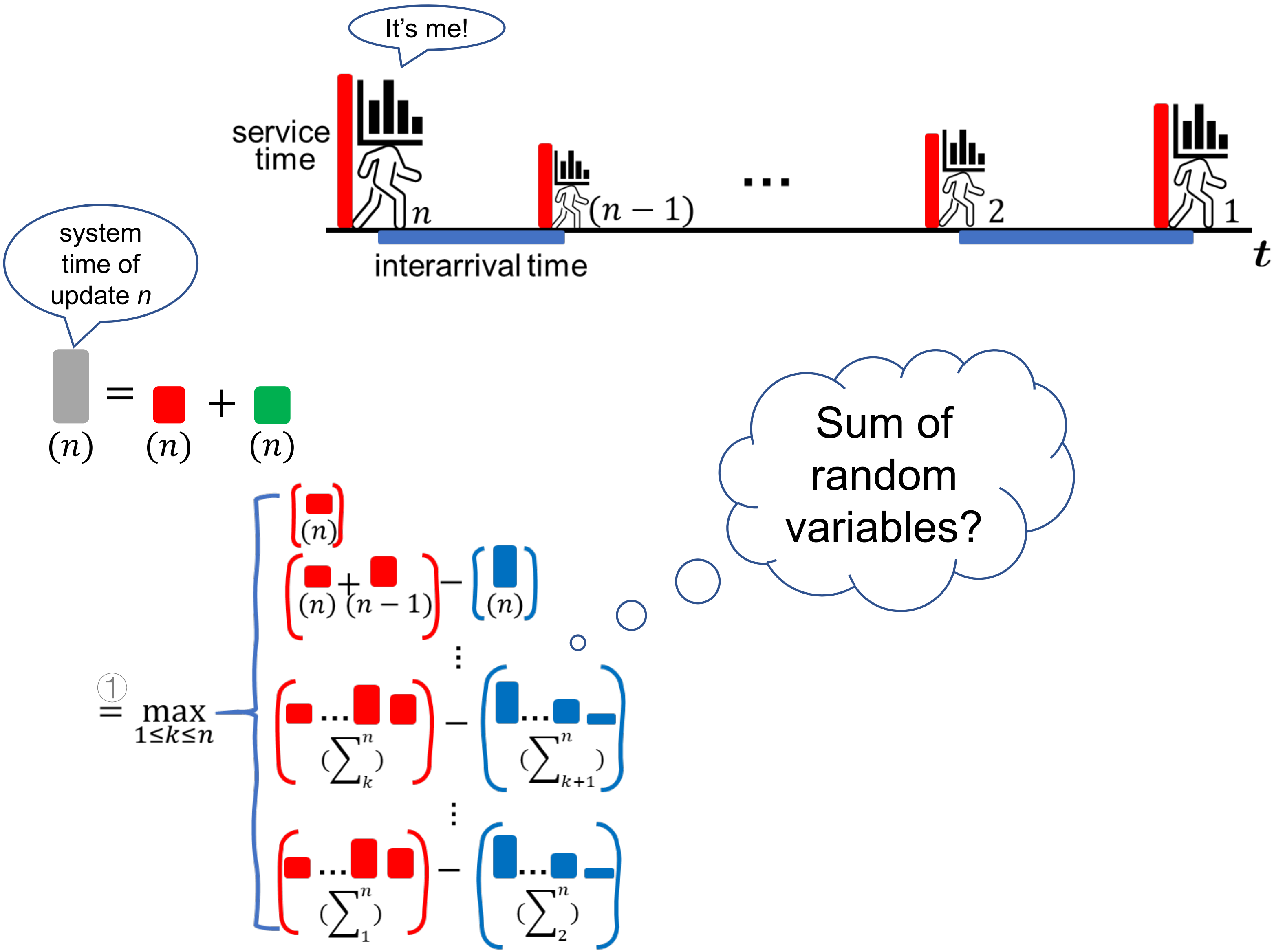

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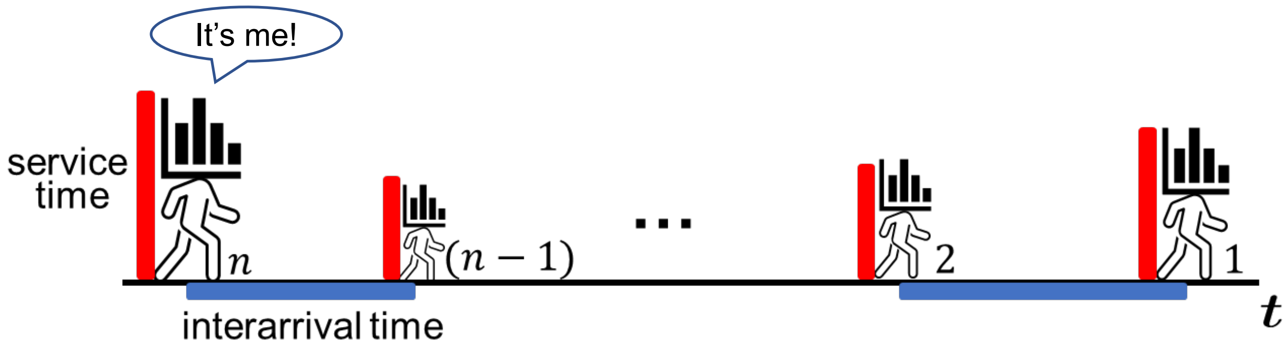

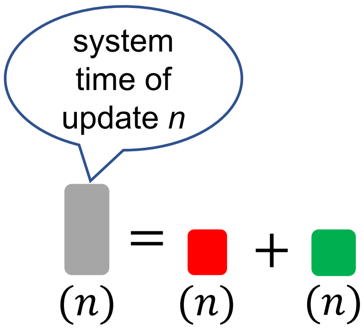

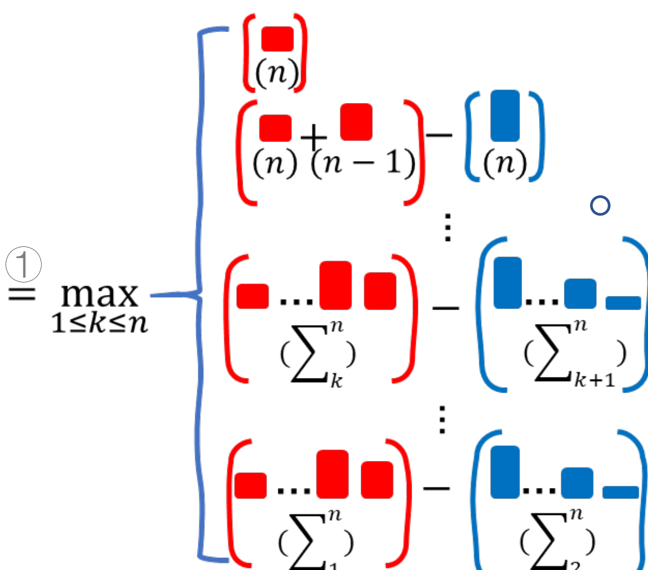

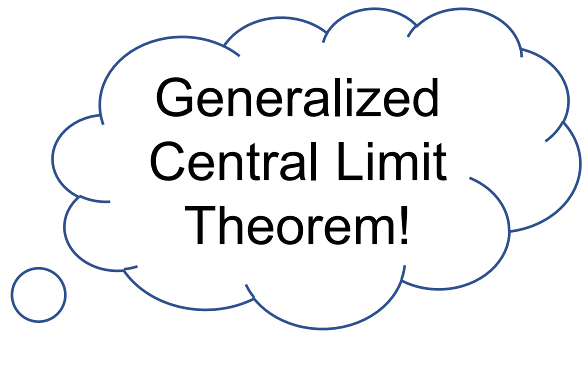

"sum of i.i.d. random variables converges to a stable distribution"

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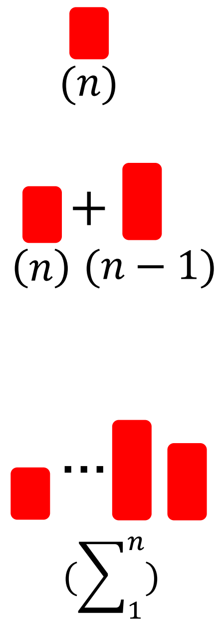
Uncertainty Set of Service Time



$$\mathcal{U}_s \triangleq \begin{cases} \le f(\Gamma_s, \alpha, \mu, n), \\ \le f(\Gamma_s, \alpha, \mu, n-1), \\ \vdots \\ \le f(\Gamma_s, \alpha, \mu, 1). \end{cases}$$

our choice

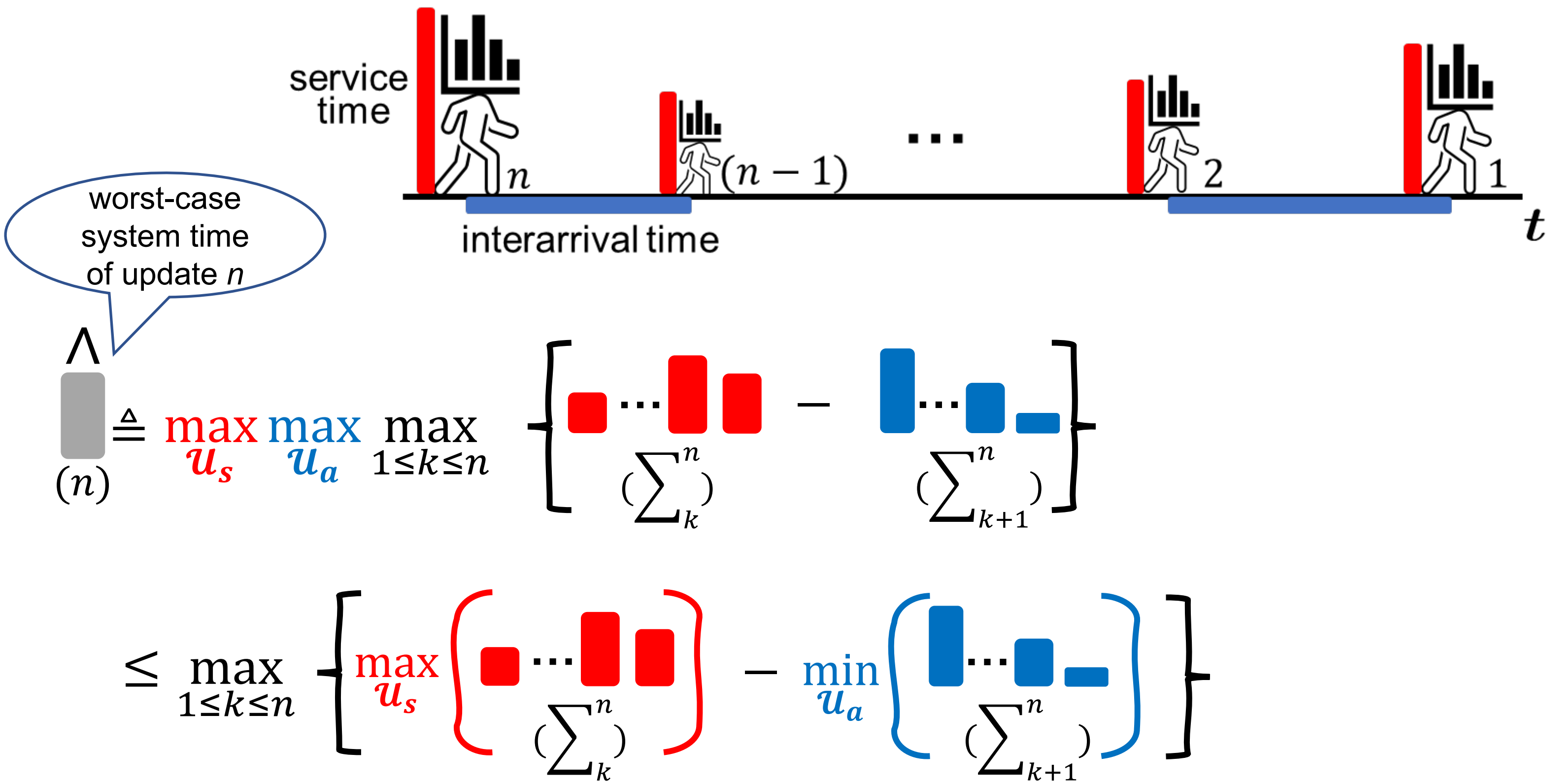
heavy-tailed

Advantages :

- More general distributions
- Non-i.i.d. service process
- Heavy-tailed distributions

$$f(\Gamma_s, \alpha, \mu, k) = \Gamma_s (n-k+1)^{\frac{1}{\alpha}} + (n-k+1)/\mu$$

Worst-Case System Time



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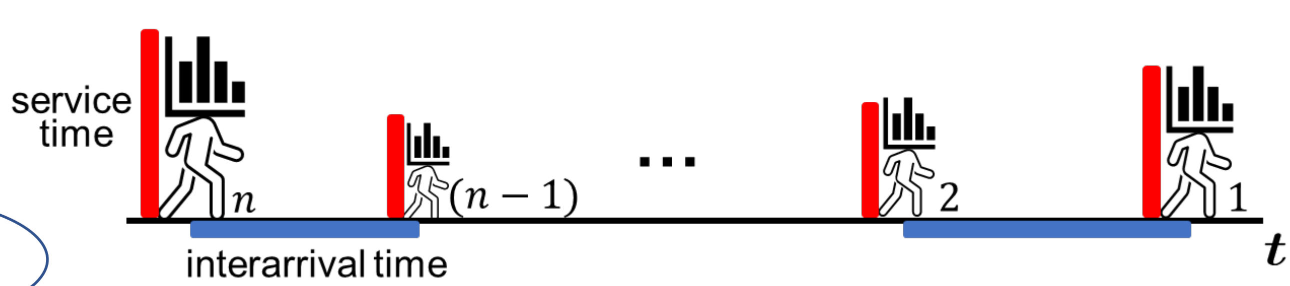

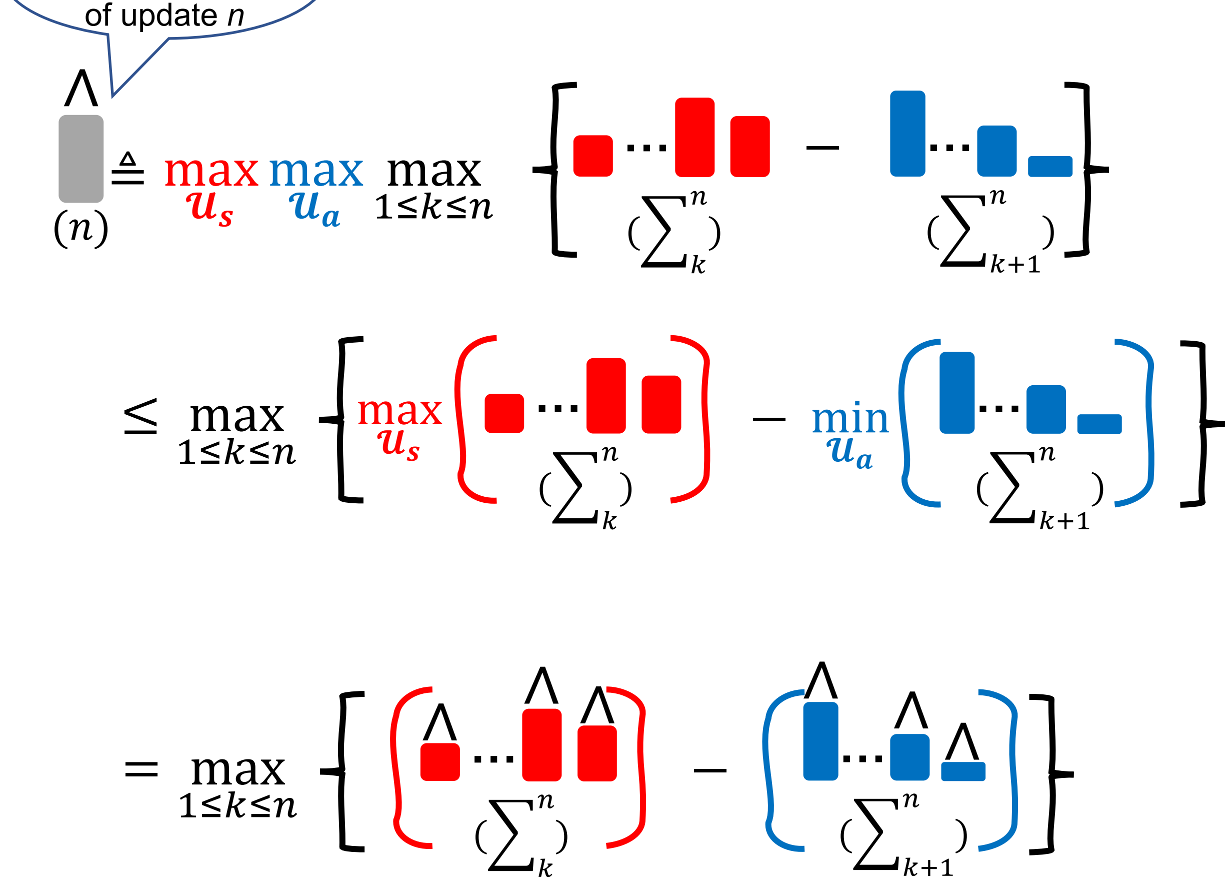

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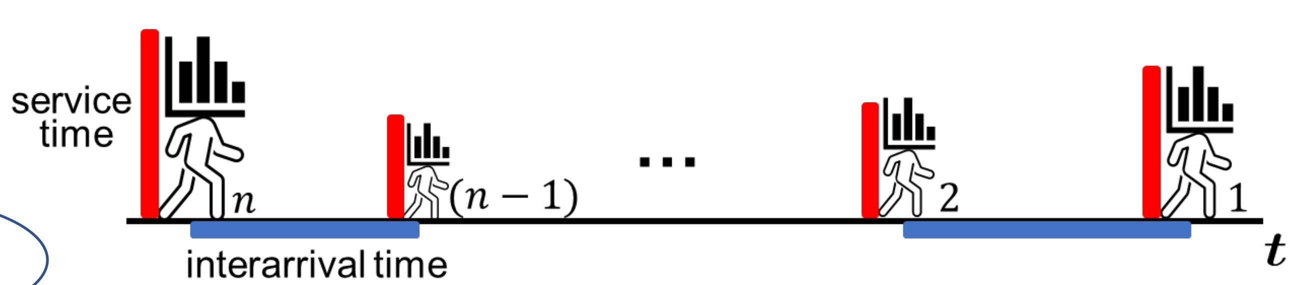

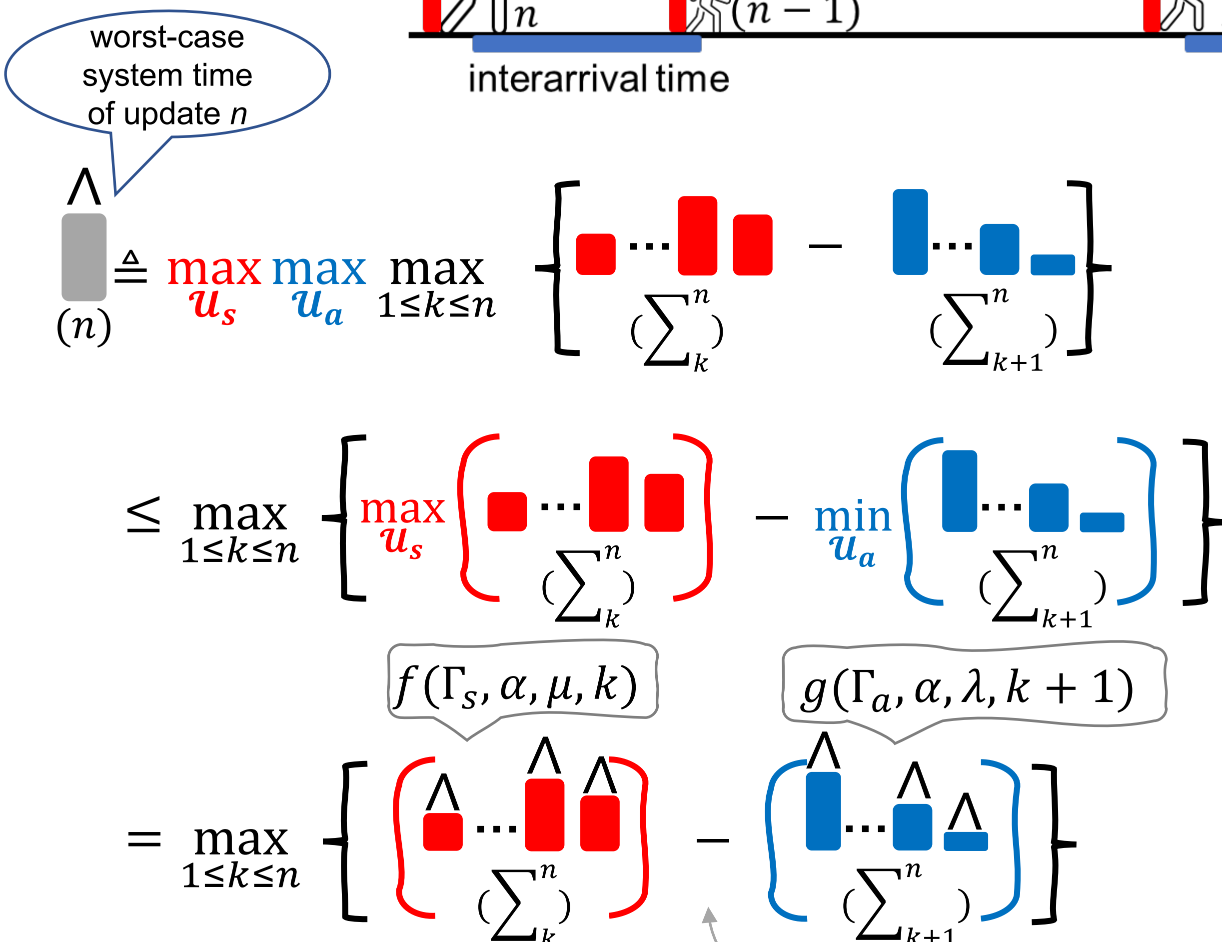

$$(n-k+1)/\mu - (n-k)/\lambda + \Gamma_s(n-k+1)^{\frac{1}{\alpha}} + \Gamma_a(n-k)^{\frac{1}{\alpha}}$$

Worst-Case System Time

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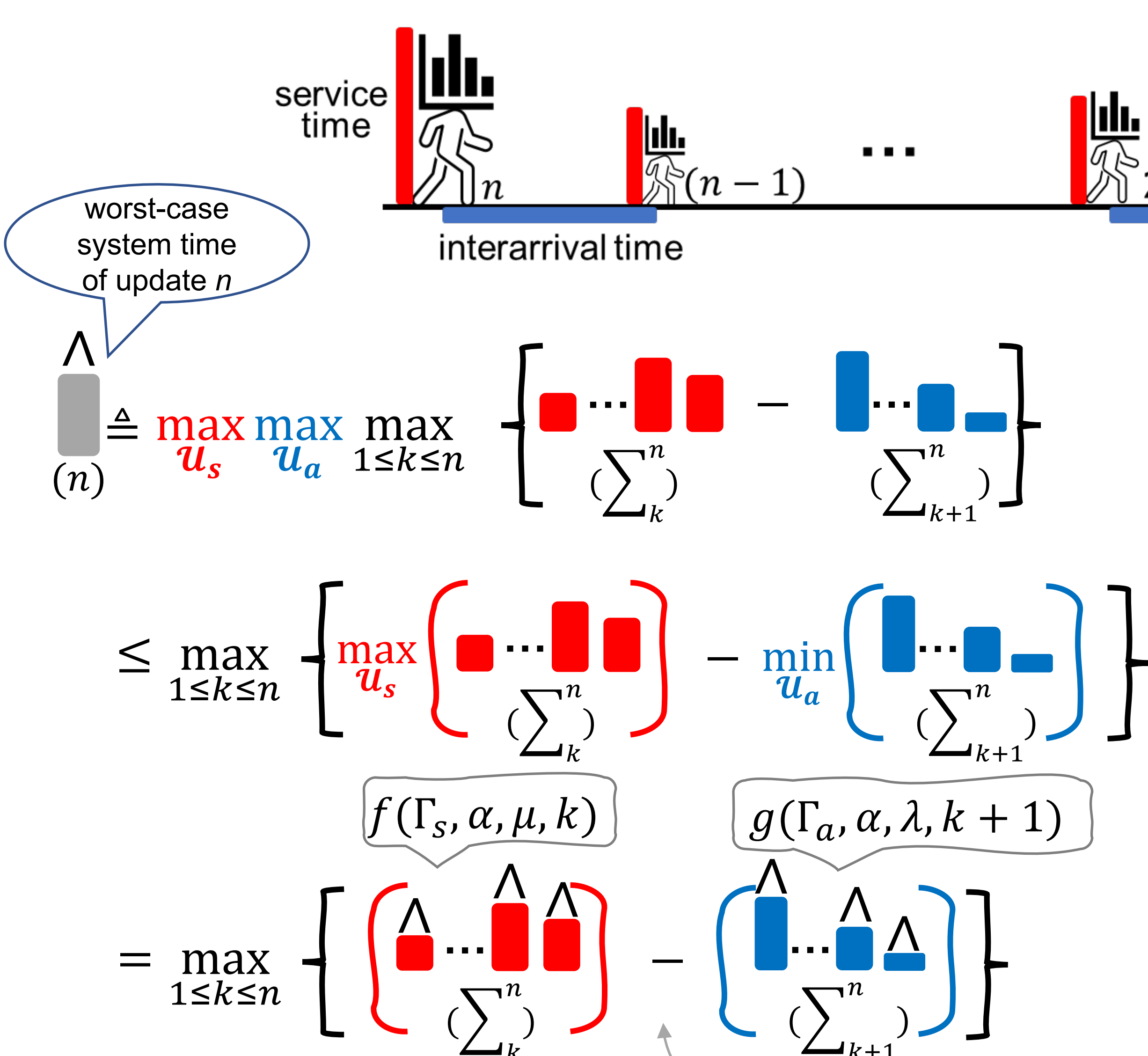
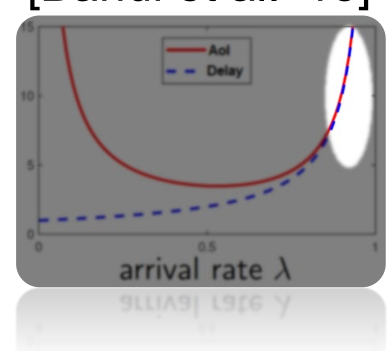
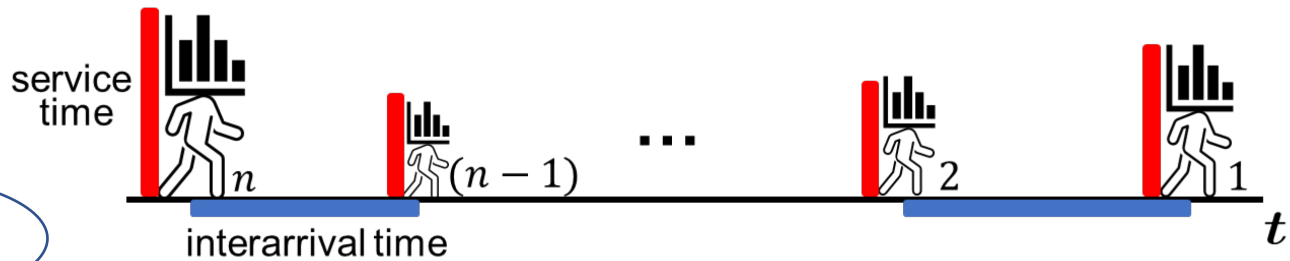


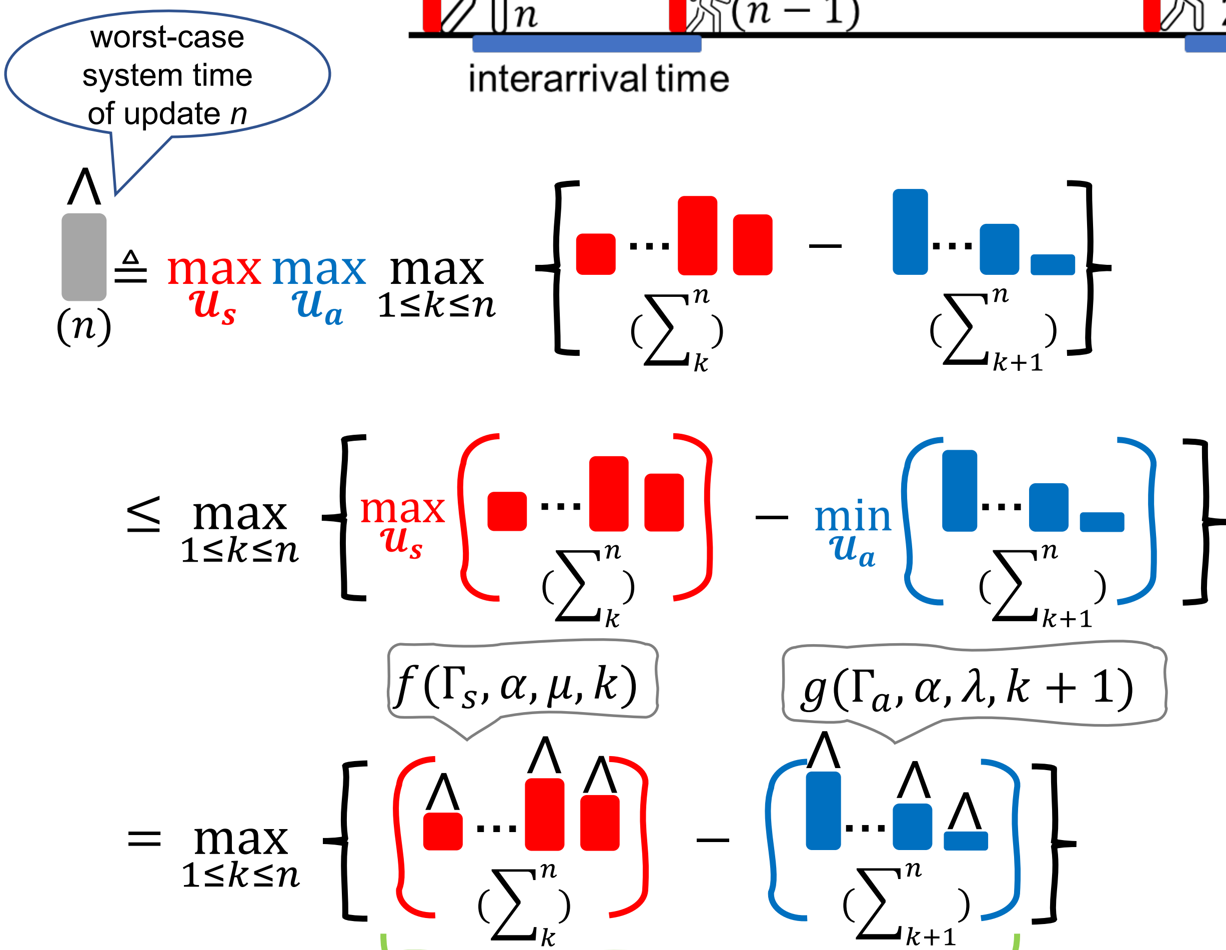
[Bandi et al. '18]

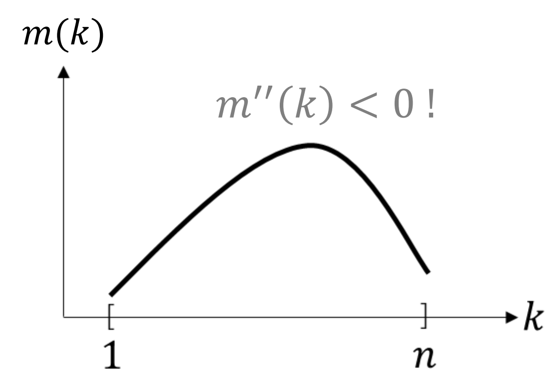


$$(n-k+1)/\mu-(n-k)/\lambda+\Gamma_s(n-k+1)^{\frac{1}{\alpha}}+\Gamma_a(n-k)^{\frac{1}{\alpha}}$$

Worst-Case System Time







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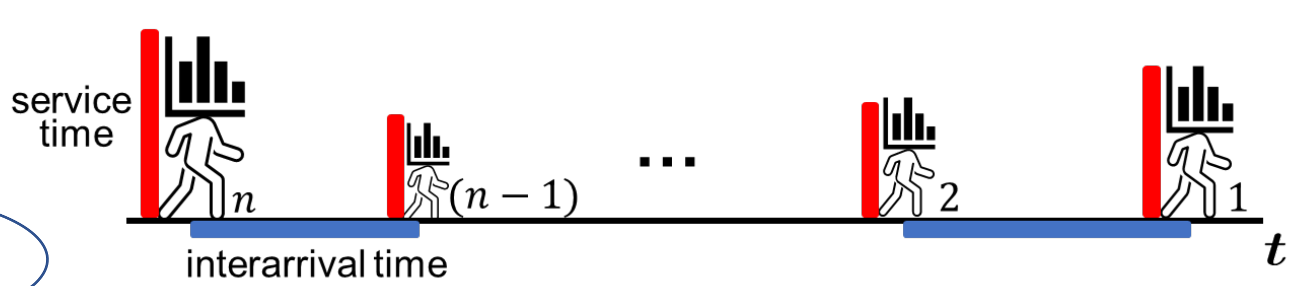

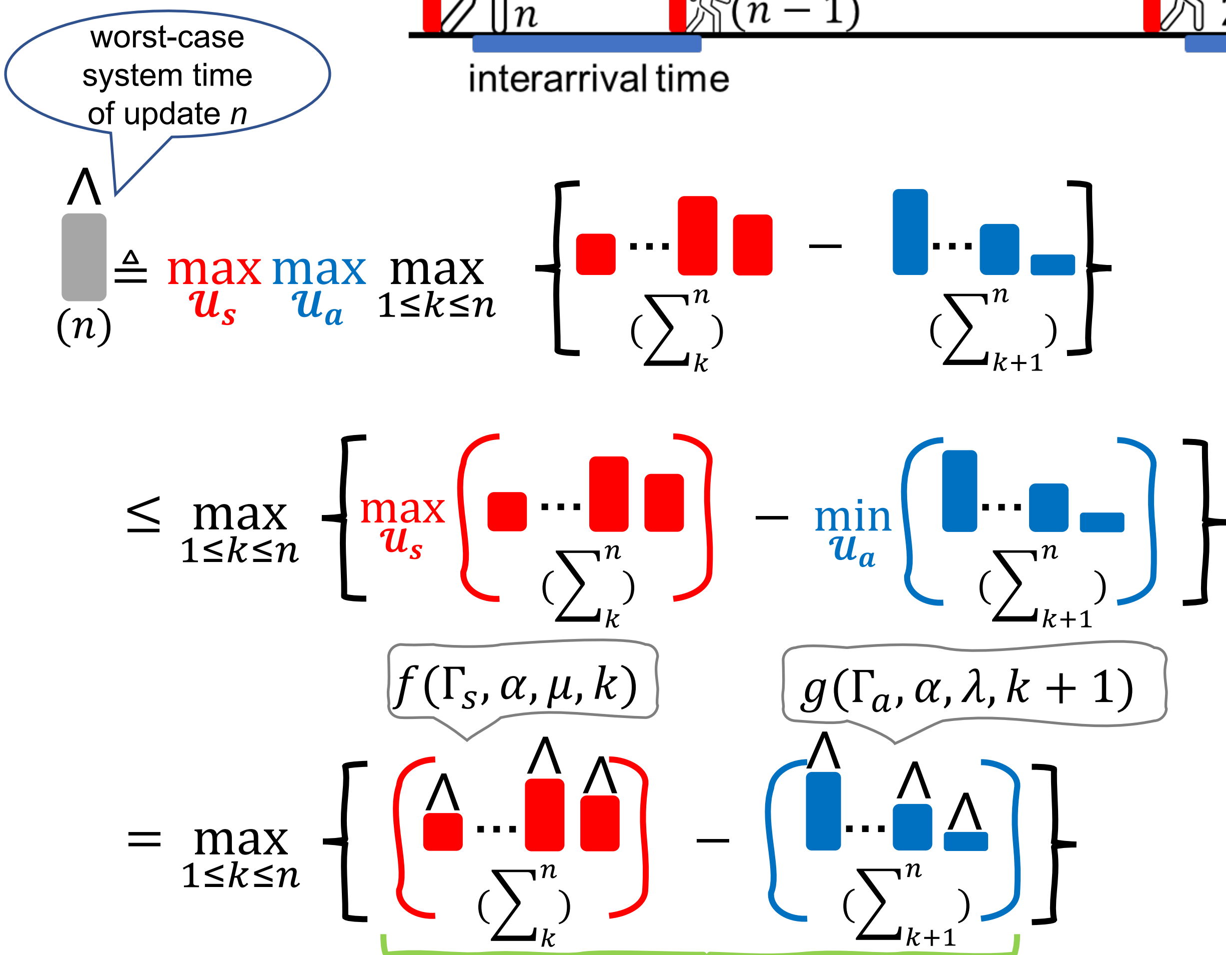

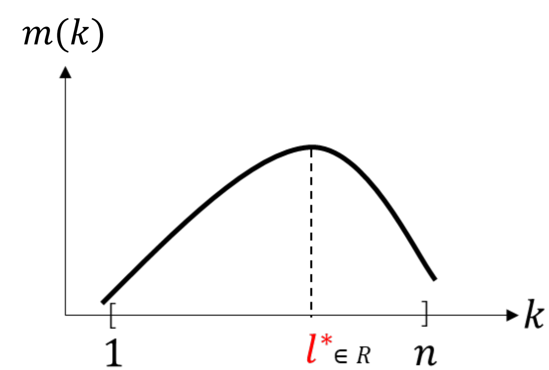

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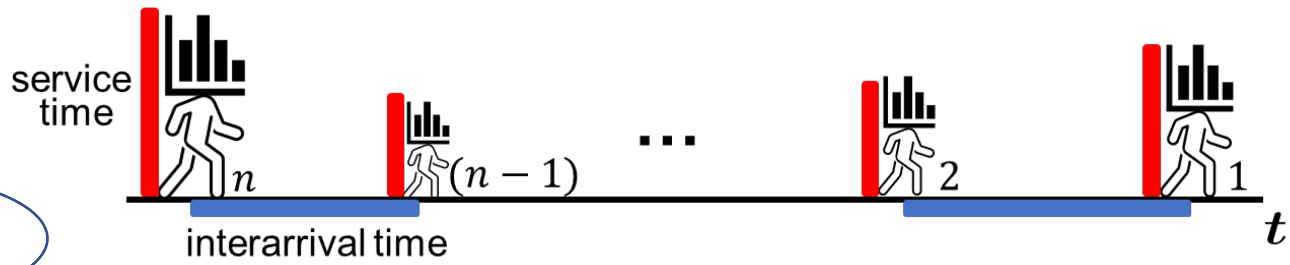

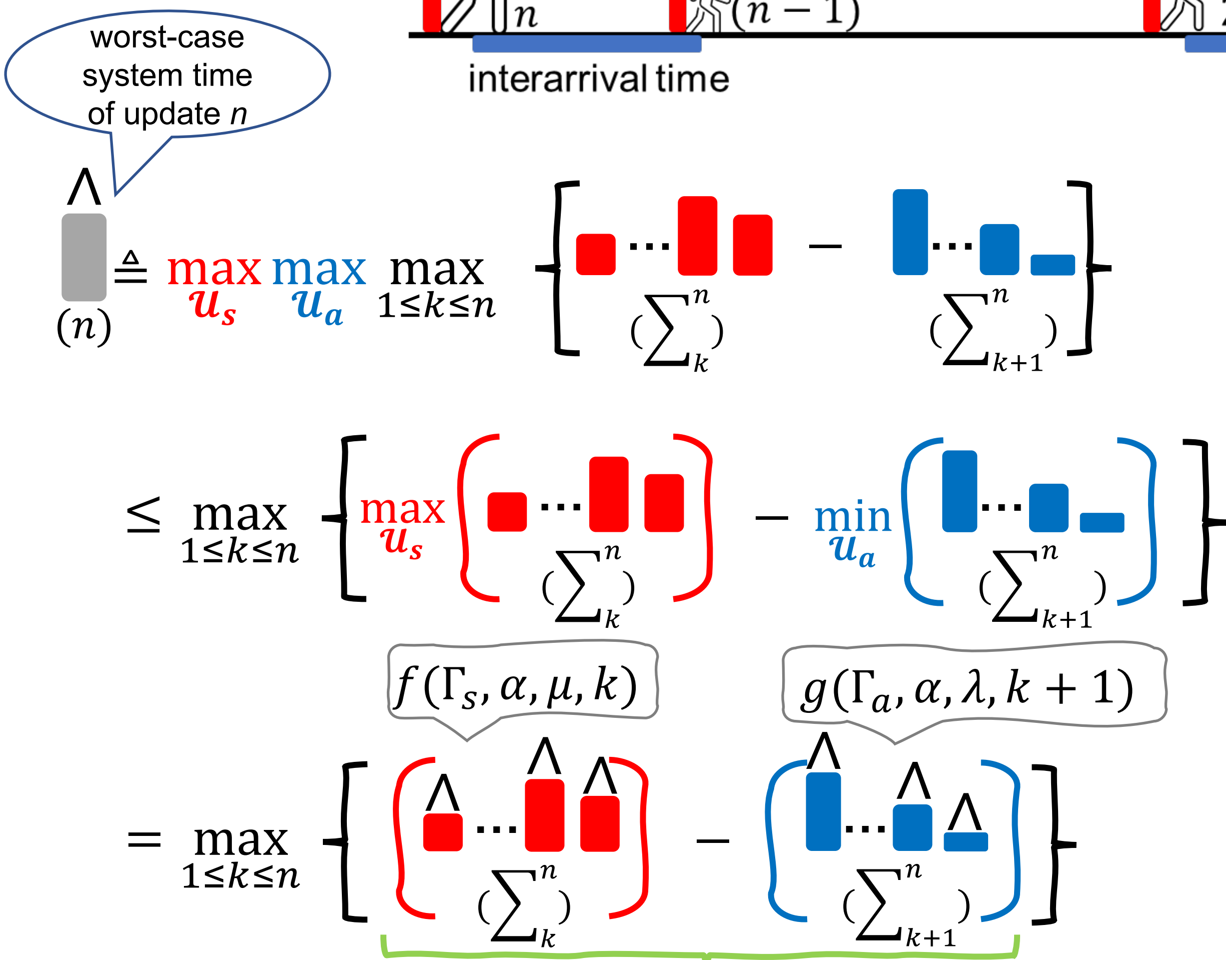

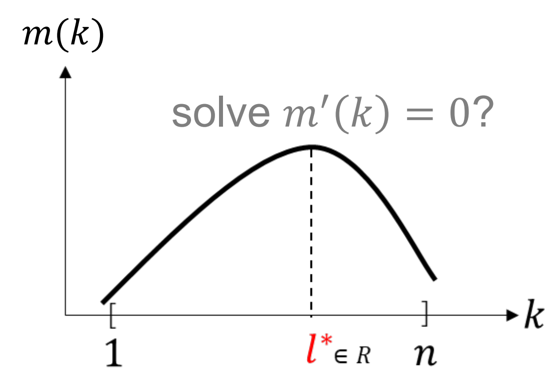

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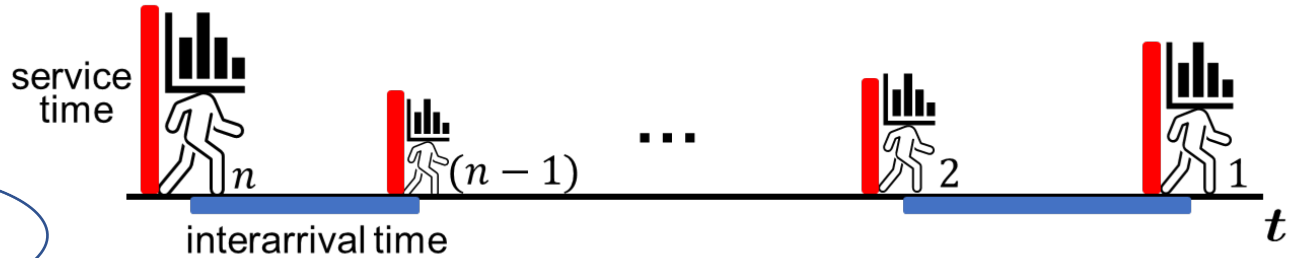
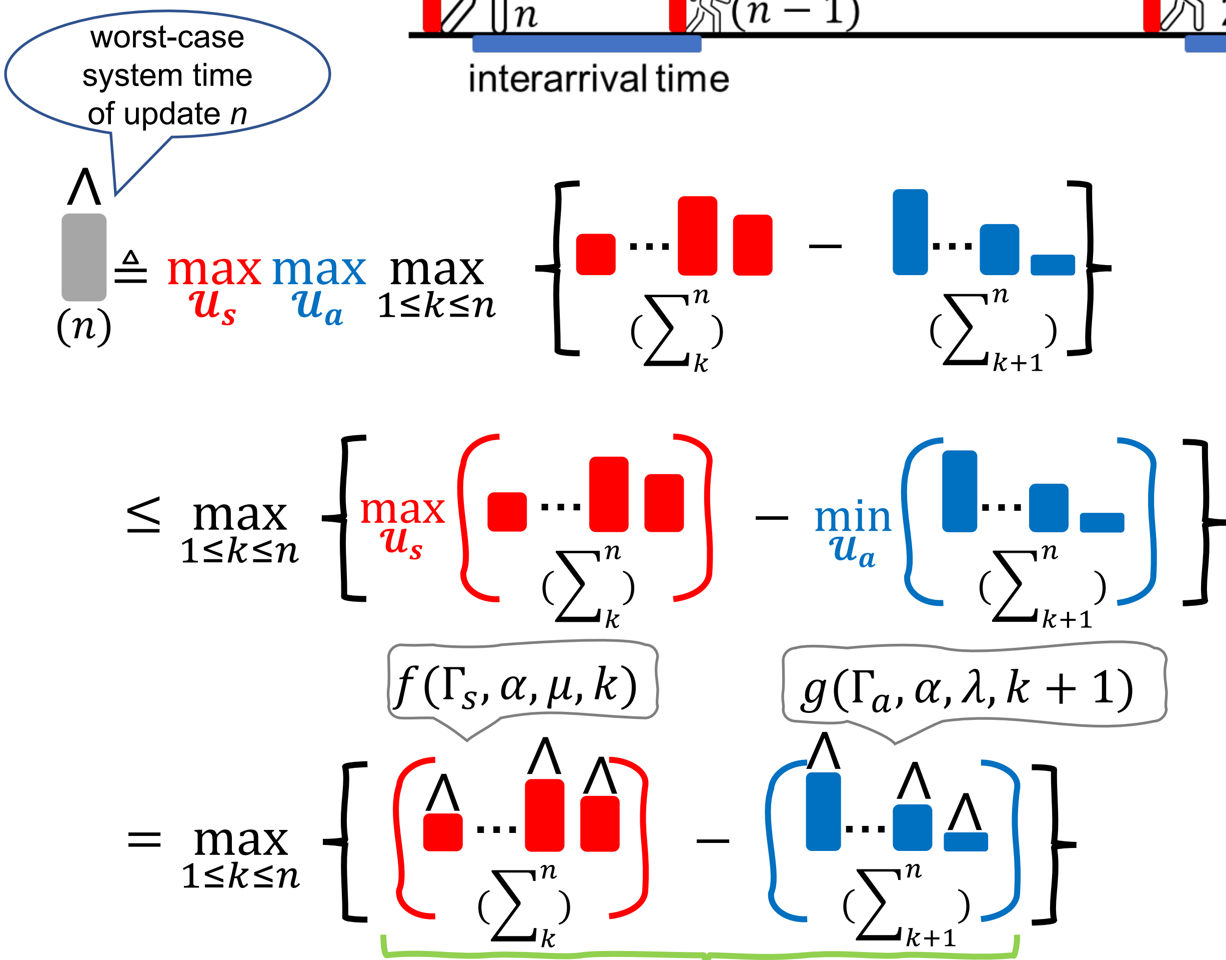
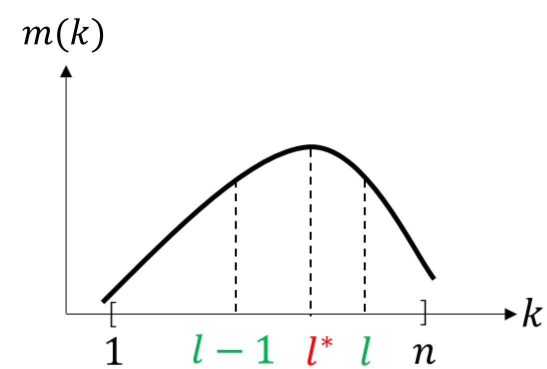
Worst-Case System Time



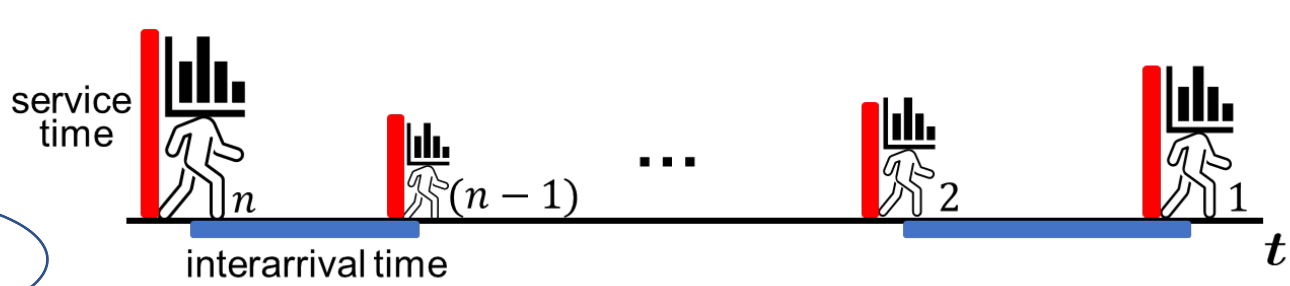


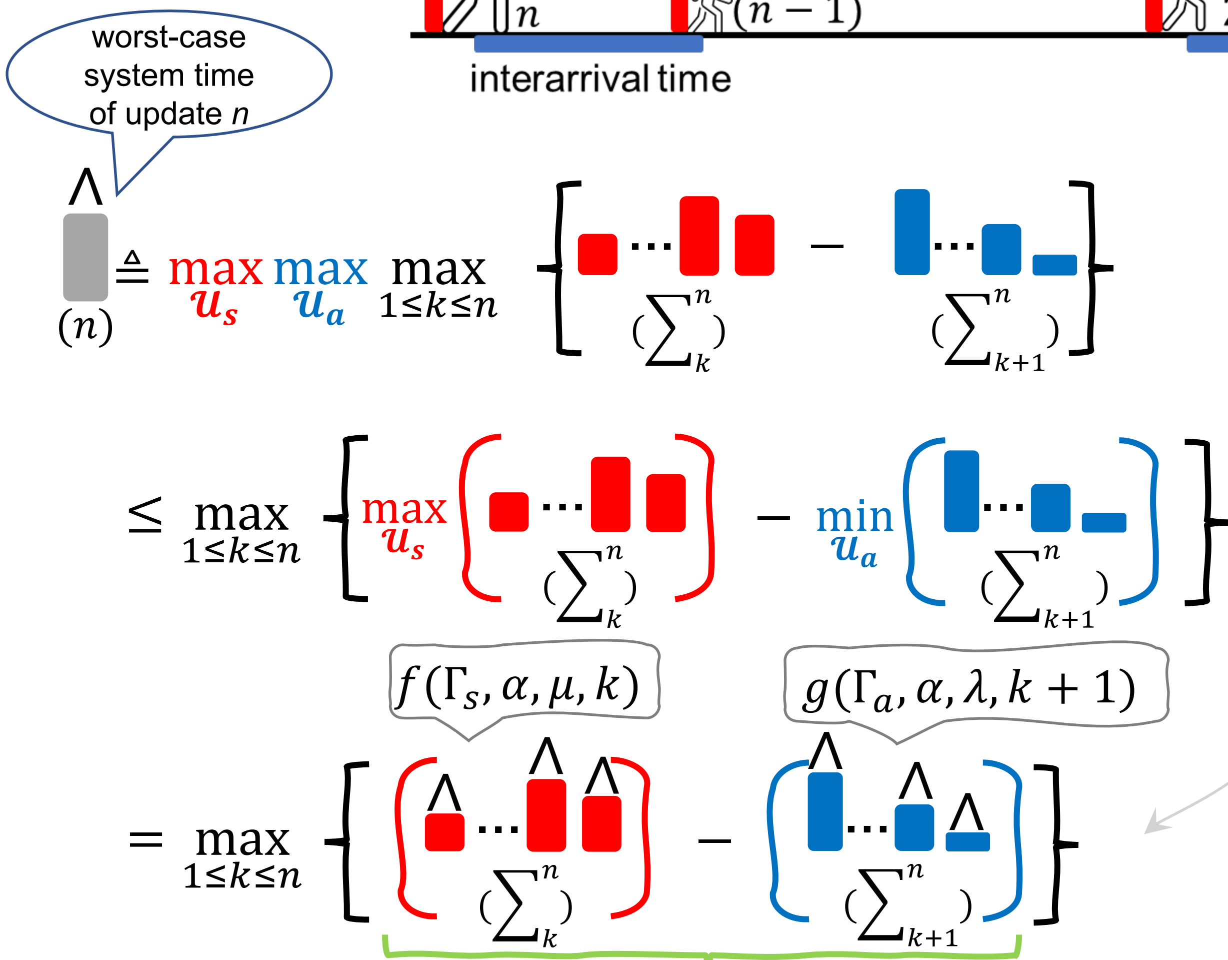
$$l = \left(\frac{\alpha(1/\lambda - 1/\mu)}{\Gamma_a + \Gamma_s}\right)^{\alpha/(1-\alpha)}$$



$$m(k) = (n-k+1)/\mu - (n-k)/\lambda + \Gamma_s(n-k+1)^{\frac{1}{\alpha}} + \Gamma_a(n-k)^{\frac{1}{\alpha}}$$

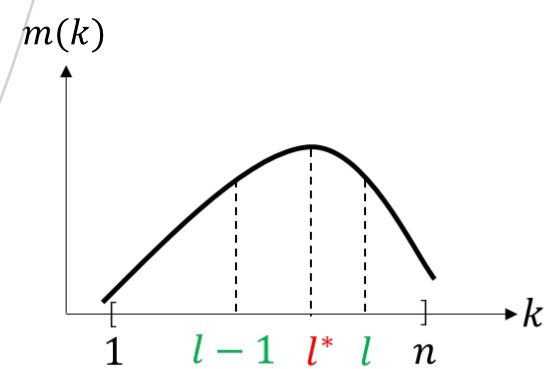
Worst-Case System Time





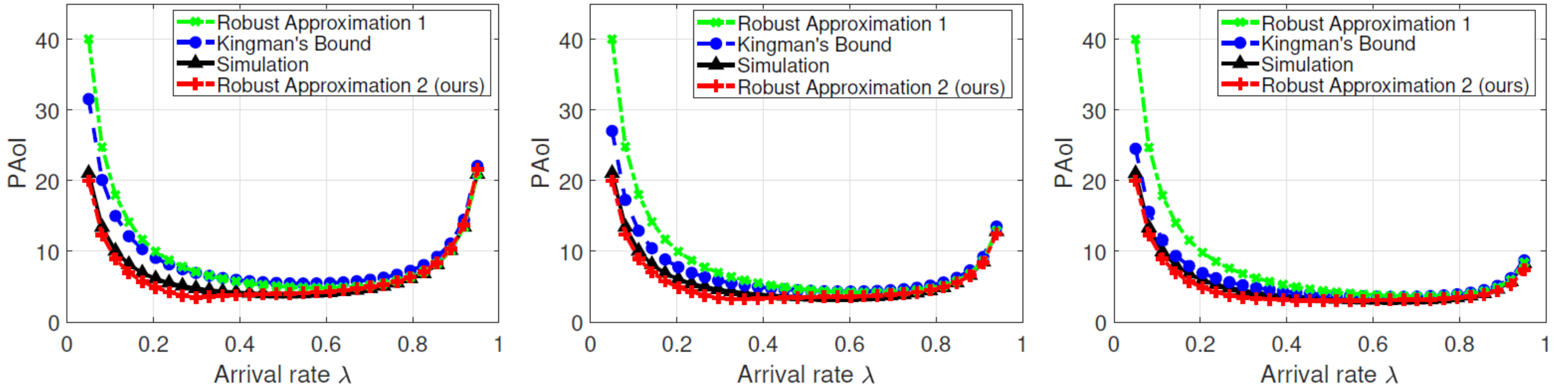
$$l = \left(\frac{\alpha(1/\lambda - 1/\mu)}{\Gamma_a + \Gamma_s}\right)^{\alpha/(1-\alpha)}$$

integral maximizer $k^* \in Z^+ \in \{\lfloor l \rfloor - 1, \lfloor l \rfloor, \lfloor l \rfloor + 1\}$



$$m(k) = (n-k+1)/\mu - (n-k)/\lambda + \Gamma_s(n-k+1)^{\frac{1}{\alpha}} + \Gamma_a(n-k)^{\frac{1}{\alpha}}$$

Numerical Result (Single-Source)

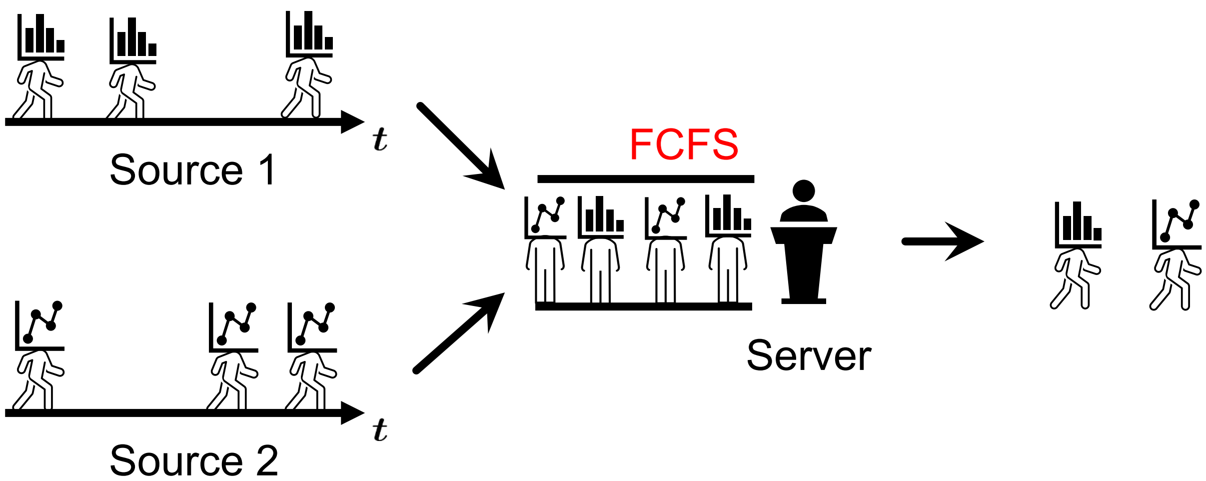


(a) Exponential distribution (b) Normal distribution (c) Uniform distribution

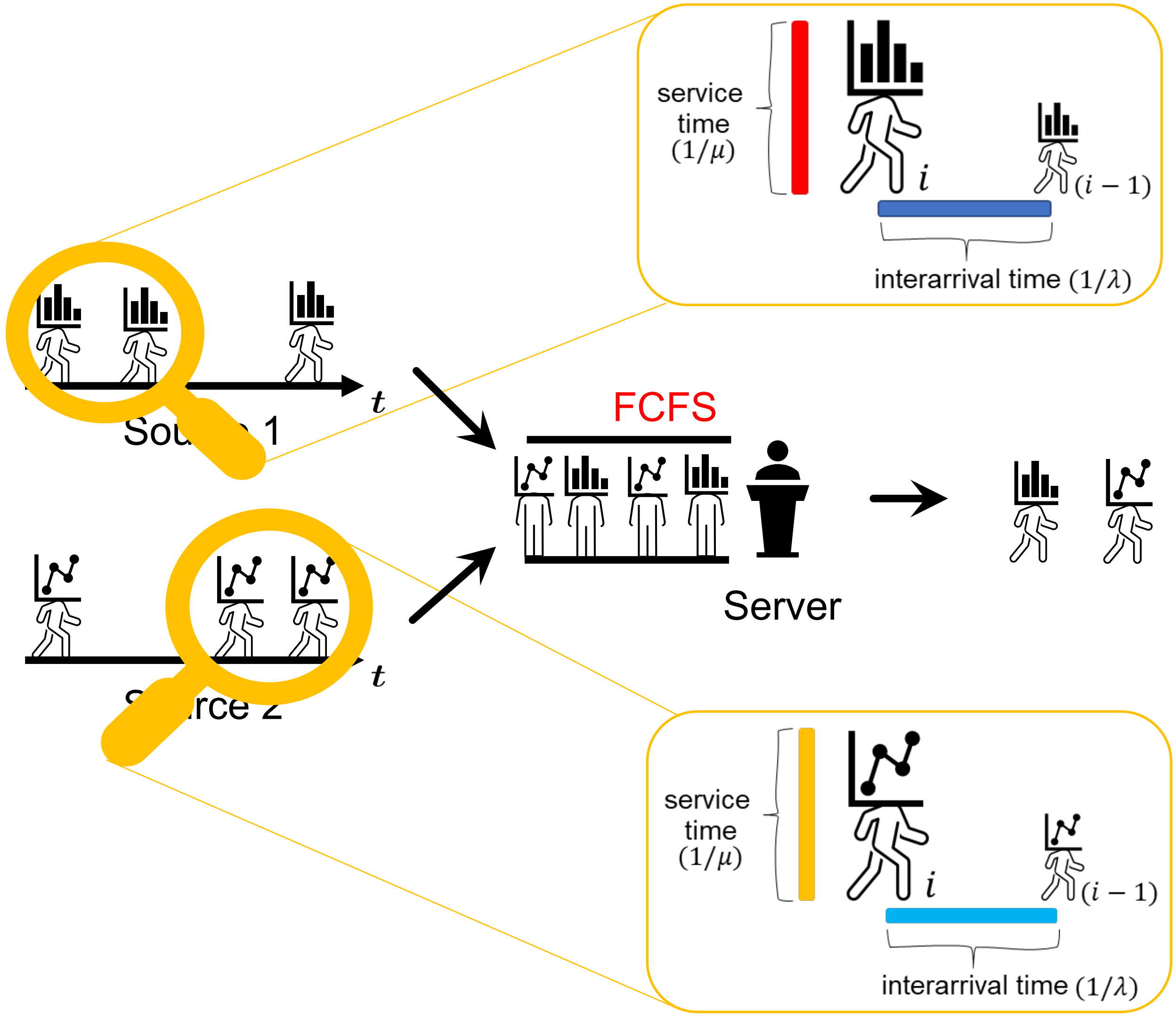
Methods	Exponential	Normal	Uniform
Kingman's bound	33.86%	22.58%	14.89%
Robust Approx. 1	32.01%	34.90%	36.49%
Robust Approx. 2 (ours)	**8.32 %**	**8.47%**	**9.28%**

Error percentage

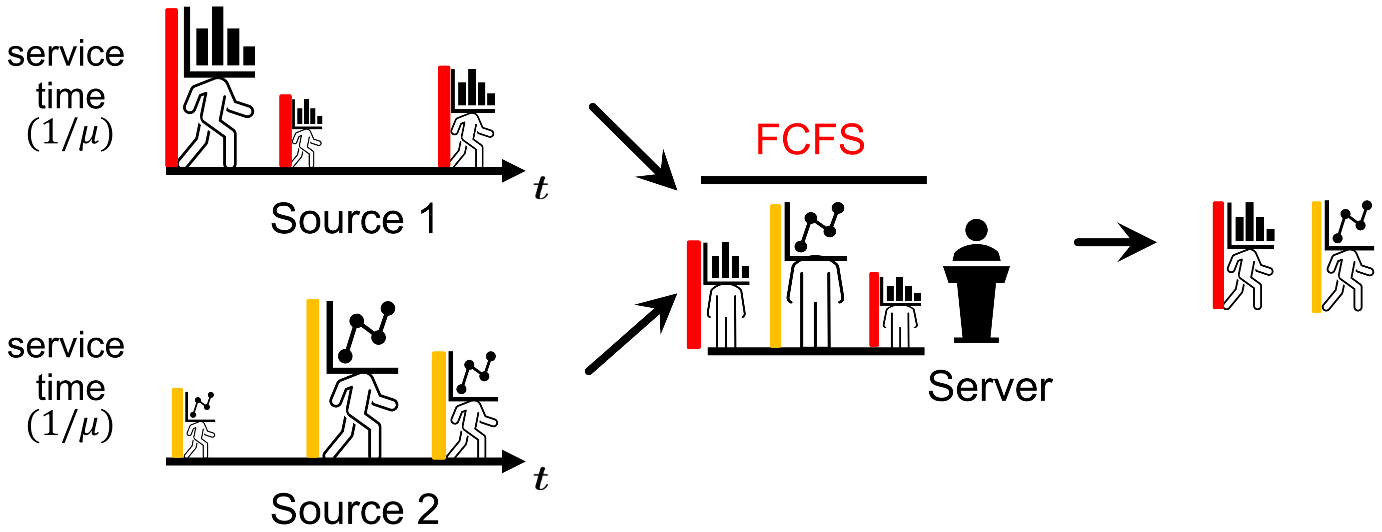
Two-Source System



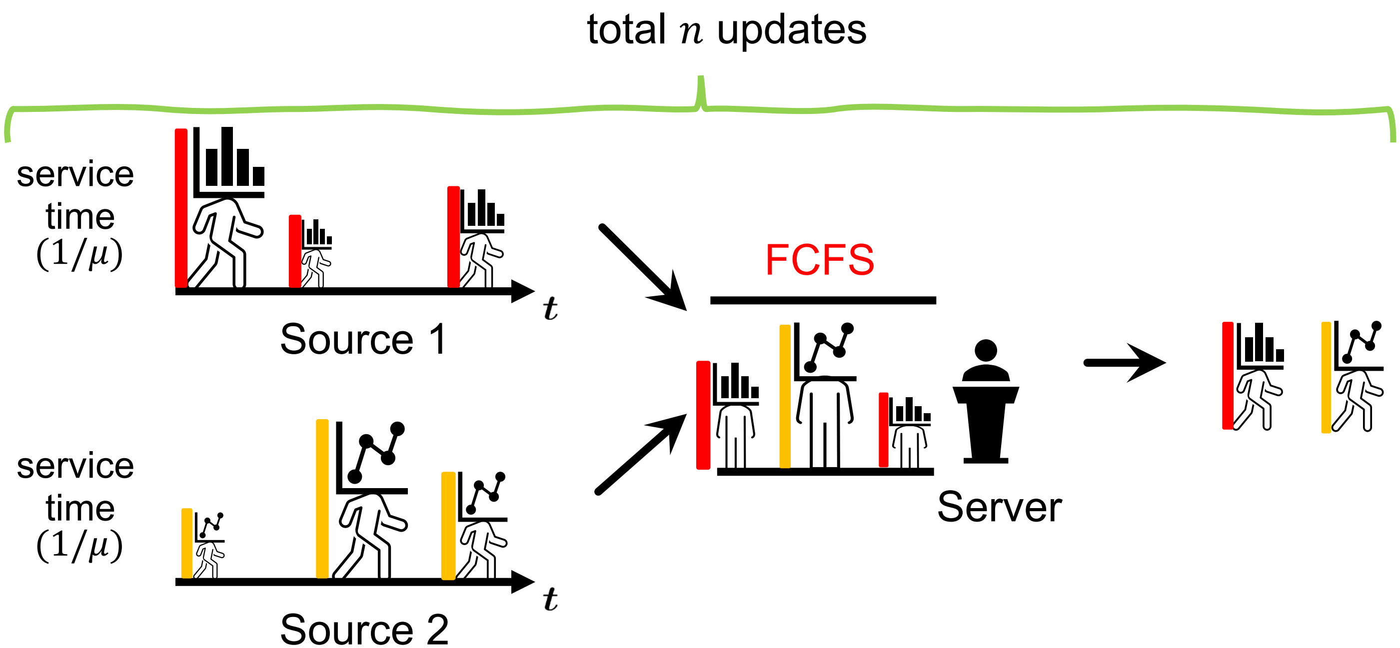
Two-Source System

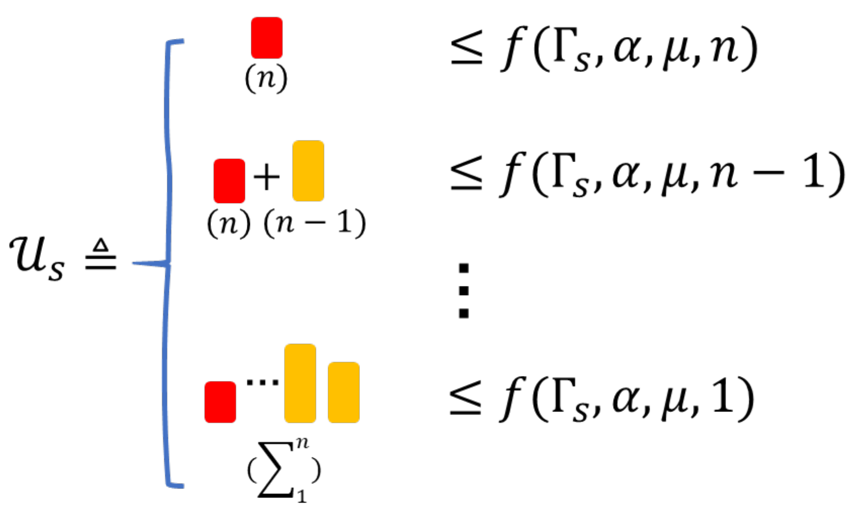


Uncertainty Set of Service Time

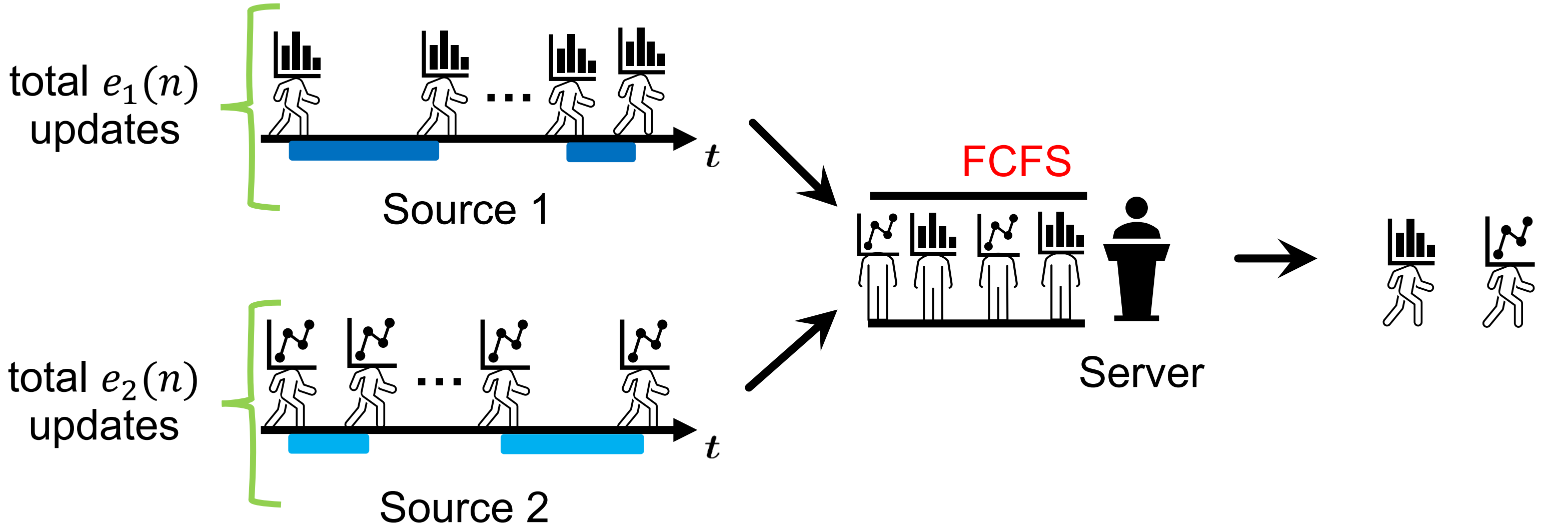


Uncertainty Set of Service Time



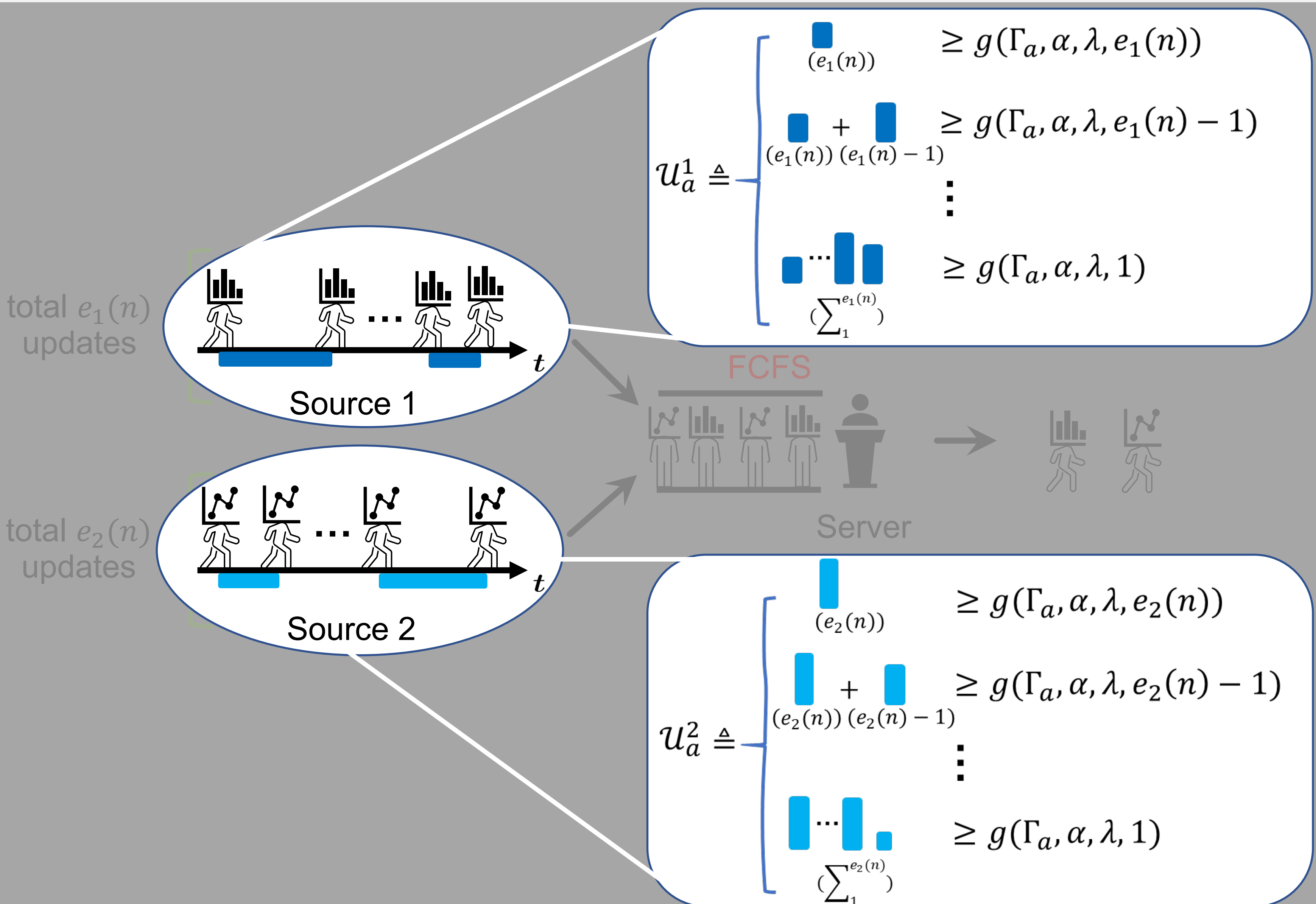


Uncertainty Sets of Interarrival Time

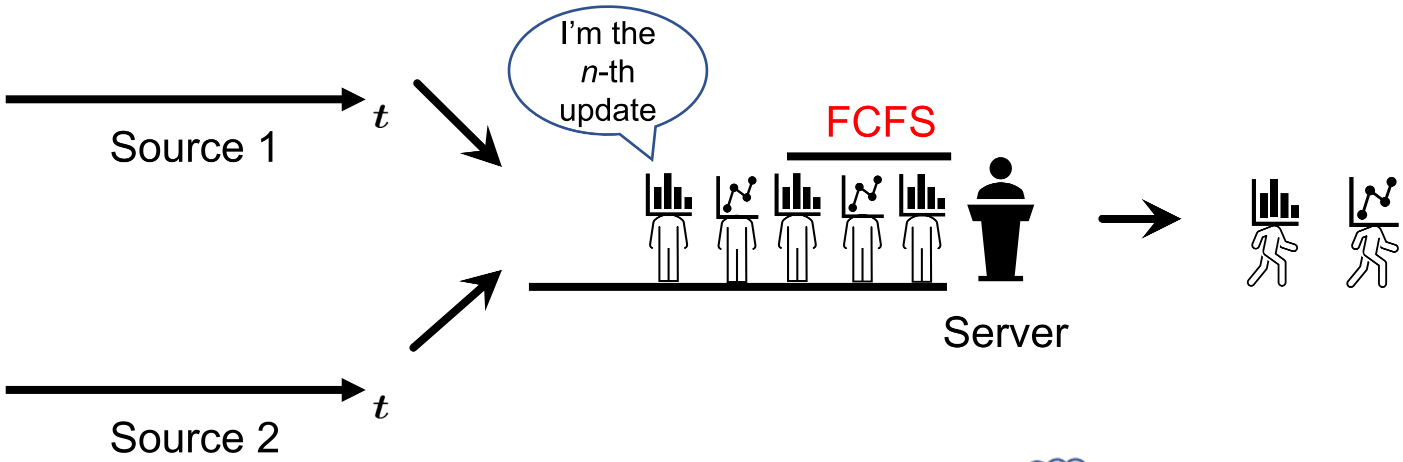


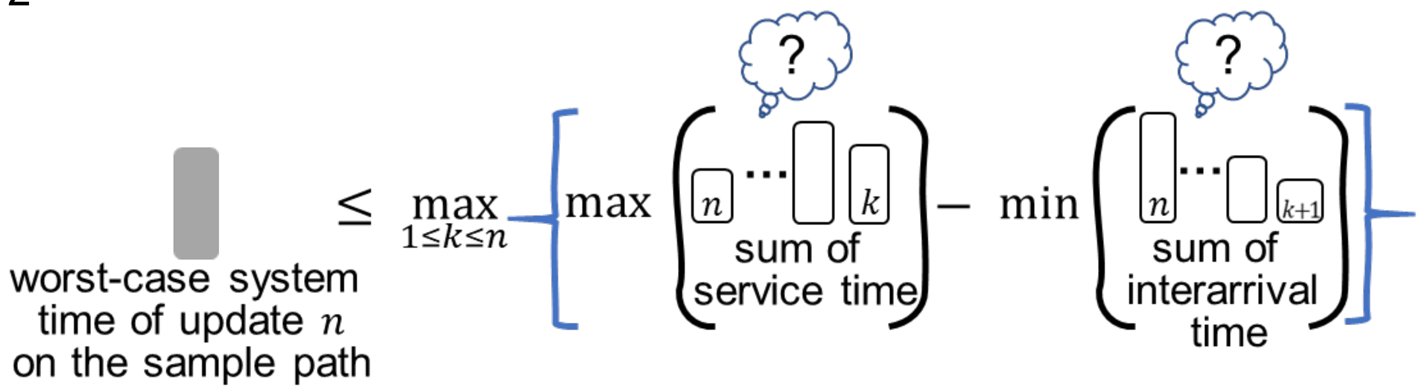
$e_1(n) + e_1(n) = \mathrm{n}$

Uncertainty Sets of Interarrival Time

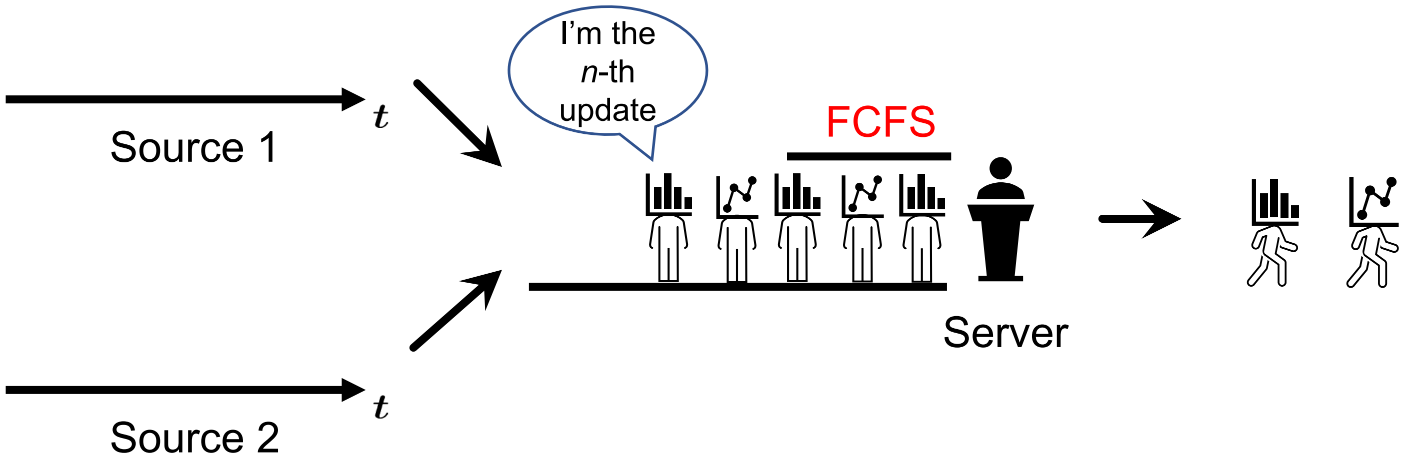


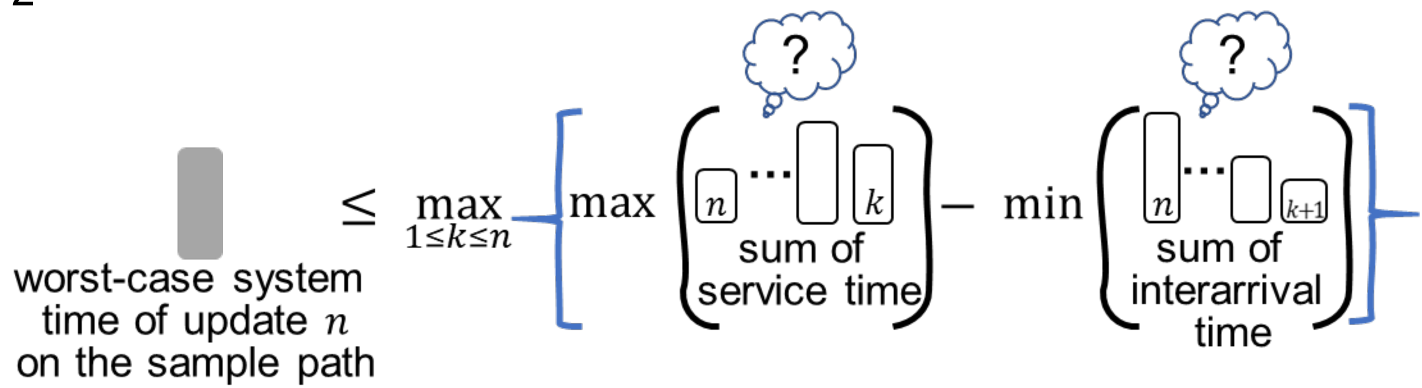
Worst-Case System Time

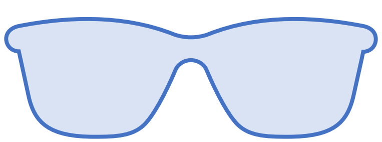




Worst-Case System Time

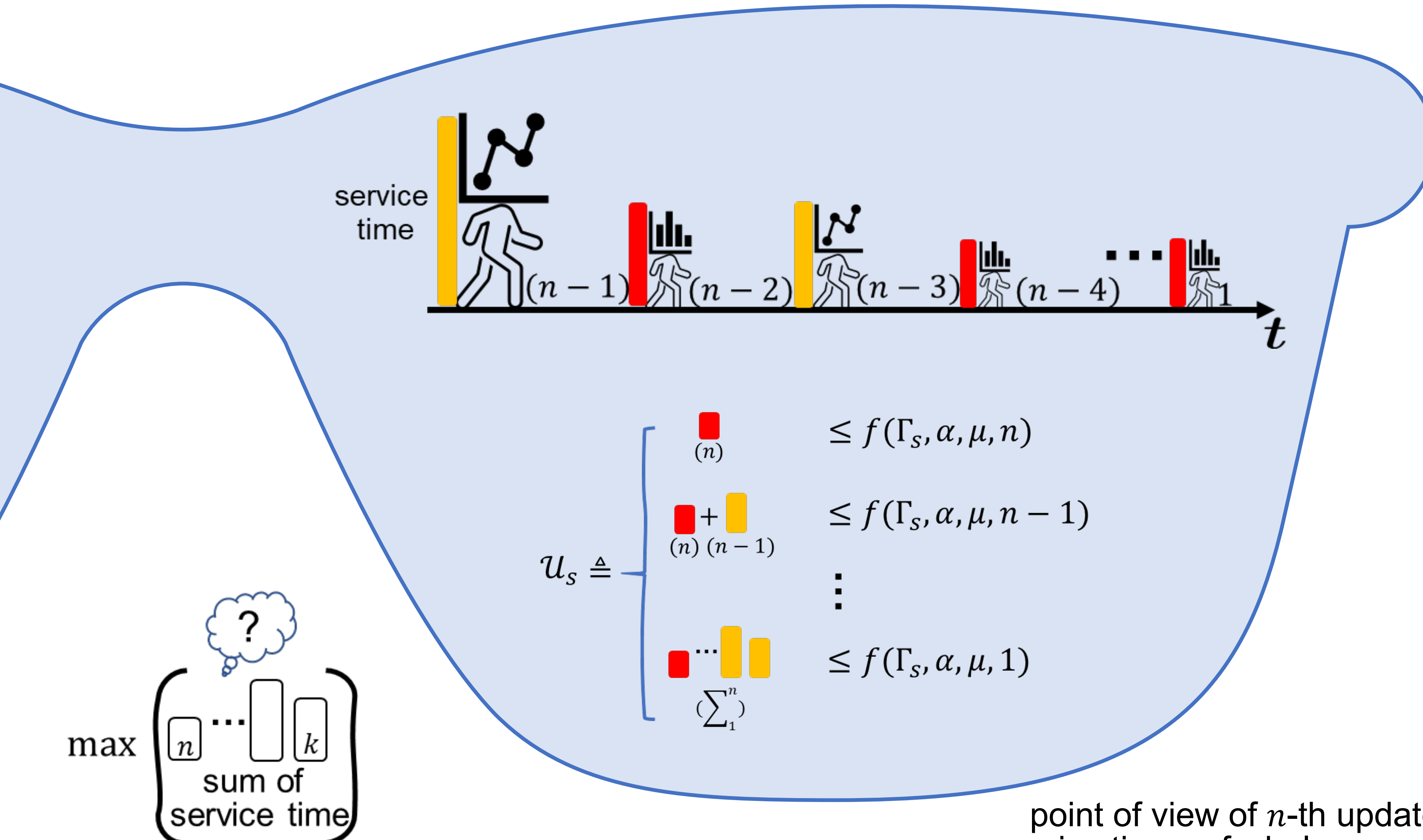






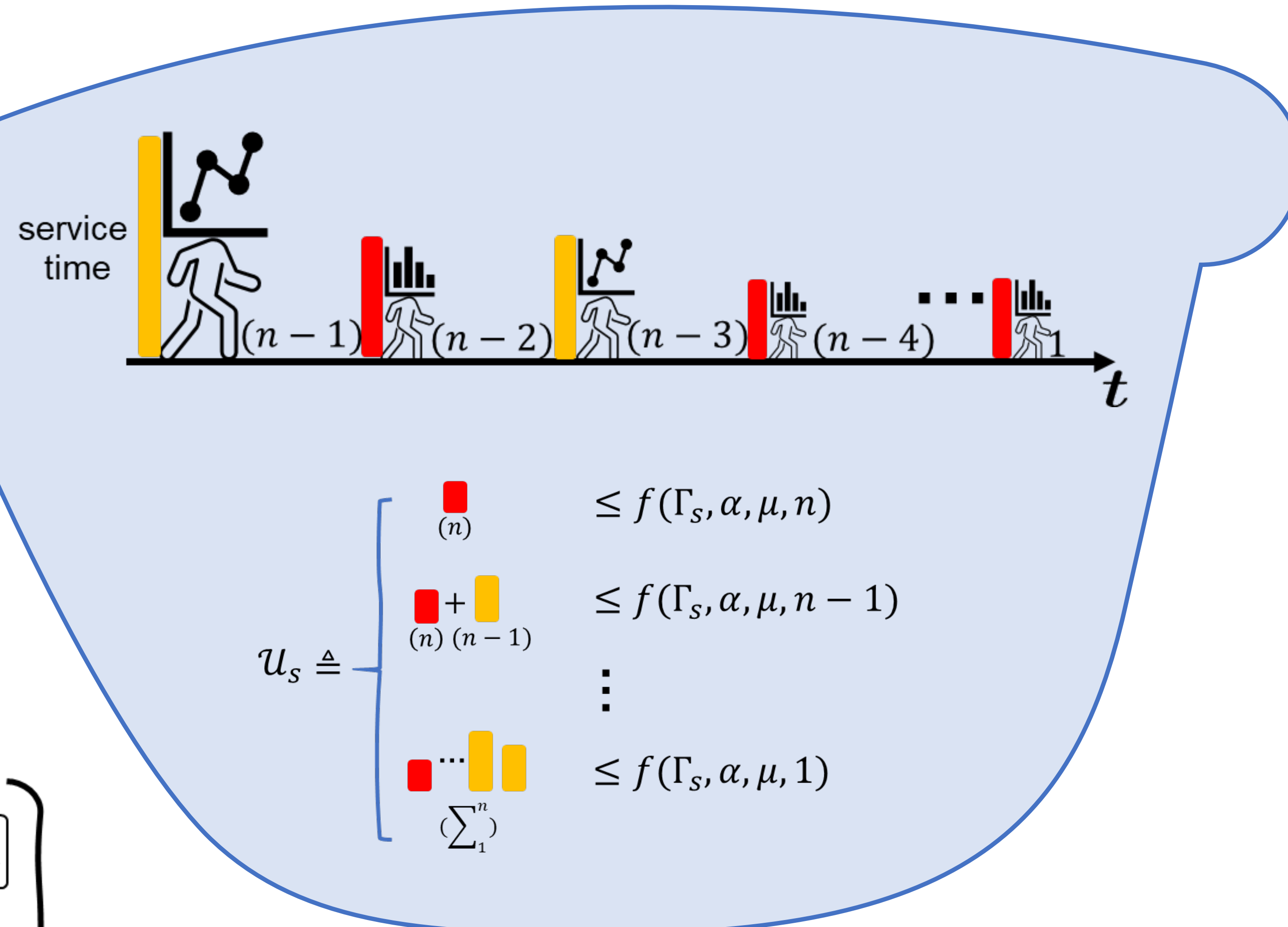
point of view of n-th update
on whole sample path

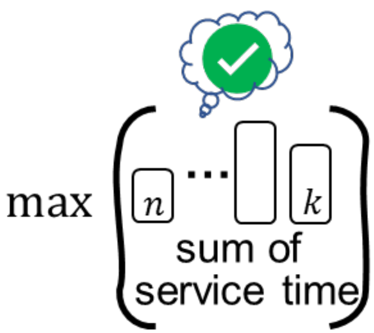
Sum of Service Time



point of view of n-th update
on service times of whole sample path

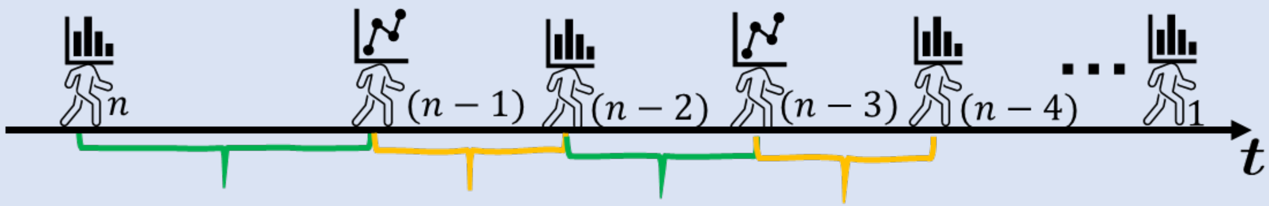
Sum of Service Time



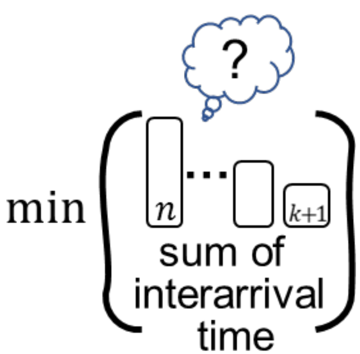


point of view of n-th update
on service times of whole sample path

Challenge: Sum of Interarrival Time

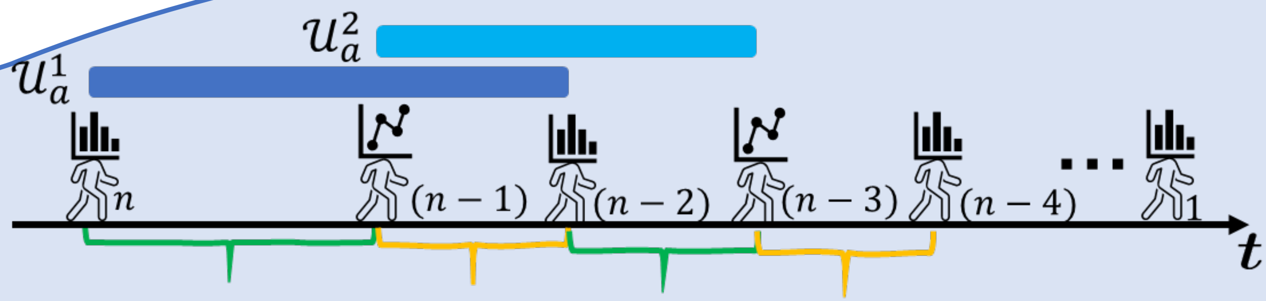


interarrival times of updates experienced **on the whole sample path**!



point of view of n-th update
on interarrival times of whole sample path

Challenge: Sum of Interarrival Time

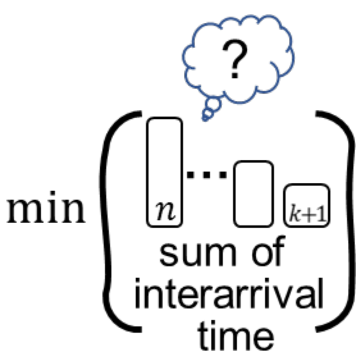


interarrival times of updates experienced **on the whole sample path**!

$\mathcal{U}_a^1 \triangleq$
- $(e_1(n)) \geq g(\Gamma_a, \alpha, \lambda, e_1(n))$
- $(e_1(n)) + (e_1(n)-1) \geq g(\Gamma_a, \alpha, \lambda, e_1(n)-1)$
- $\vdots$
- $(\sum_1^{e_1(n)}) \geq g(\Gamma_a, \alpha, \lambda, 1)$

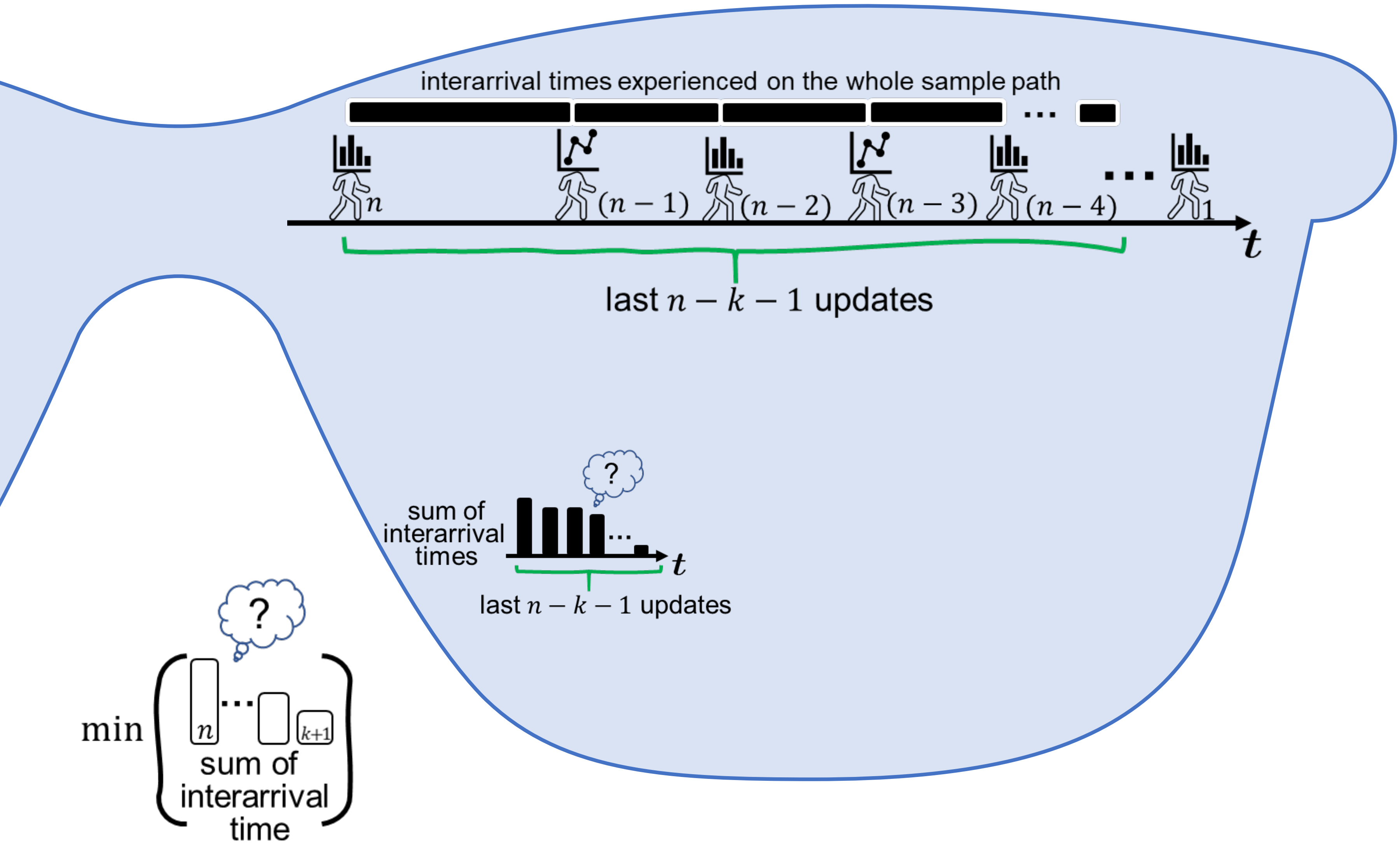
$\mathcal{U}_a^2 \triangleq$
- $(e_2(n)) \geq g(\Gamma_a, \alpha, \lambda, e_2(n))$
- $(e_2(n)) + (e_2(n)-1) \geq g(\Gamma_a, \alpha, \lambda, e_2(n)-1)$
- $\vdots$
- $(\sum_1^{e_2(n)}) \geq g(\Gamma_a, \alpha, \lambda, 1)$

our uncertainty sets of interarrival time are defined for **each source**

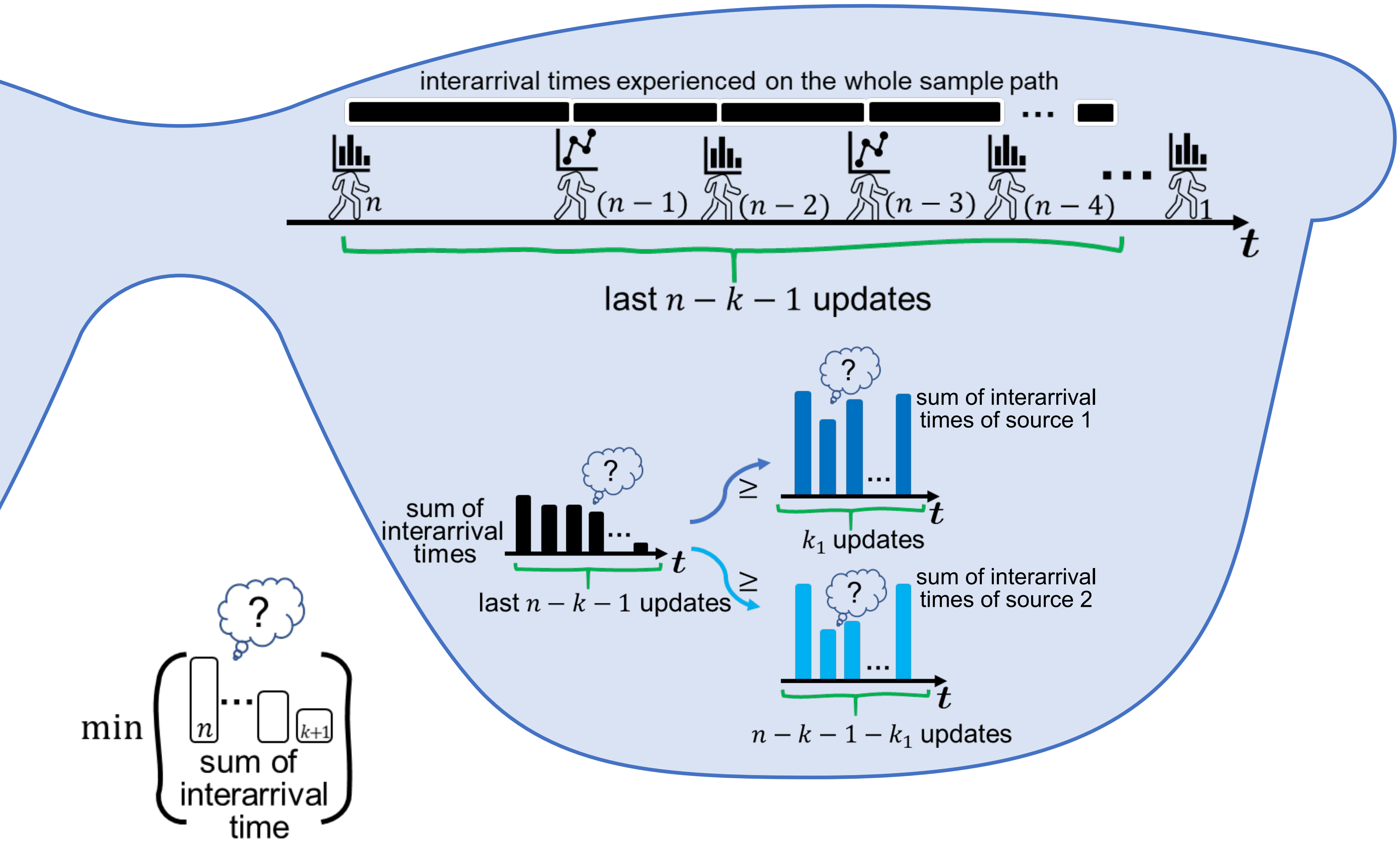


point of view of n-th update on interarrival times of whole sample path

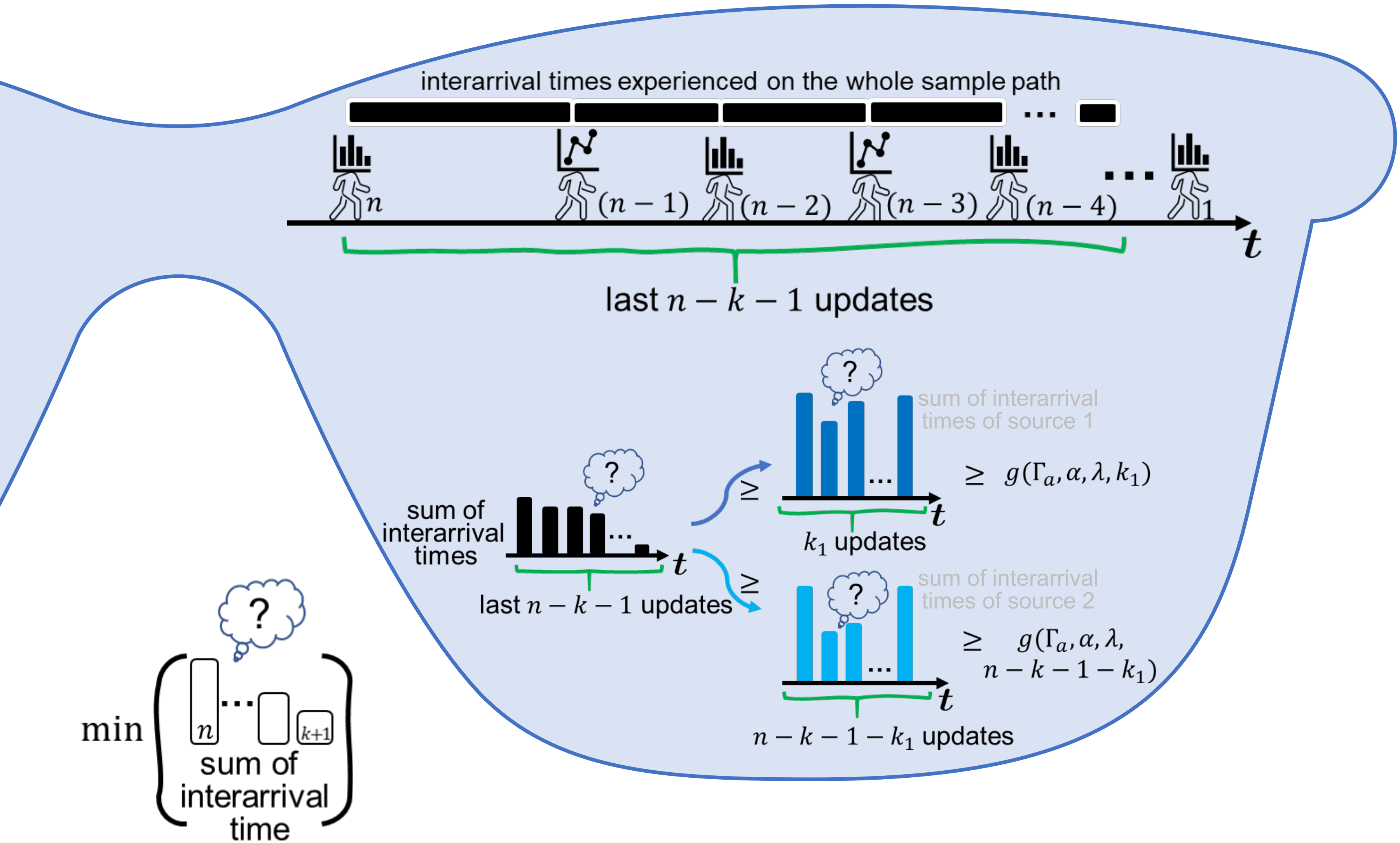
Our Solution



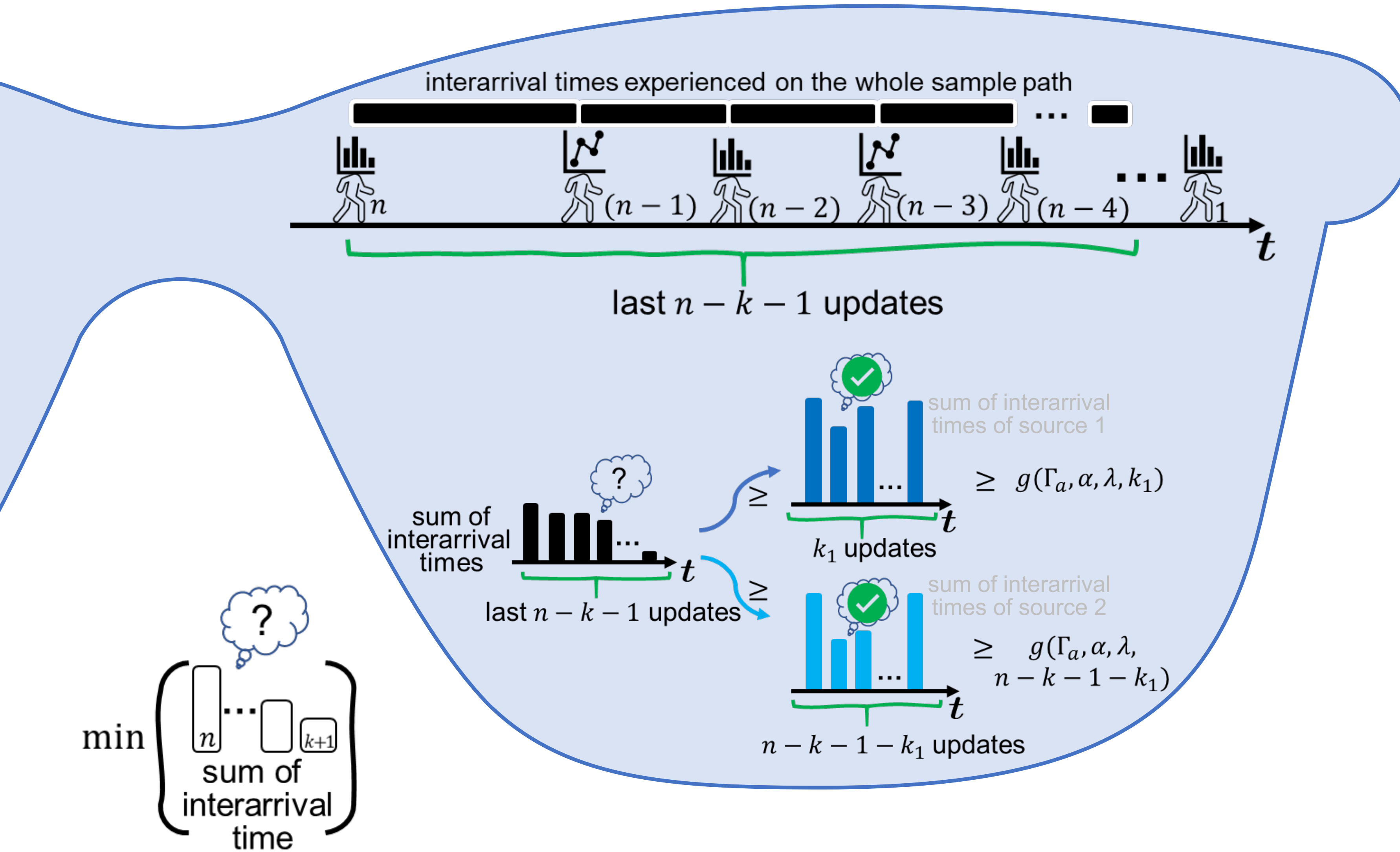
Our Solution



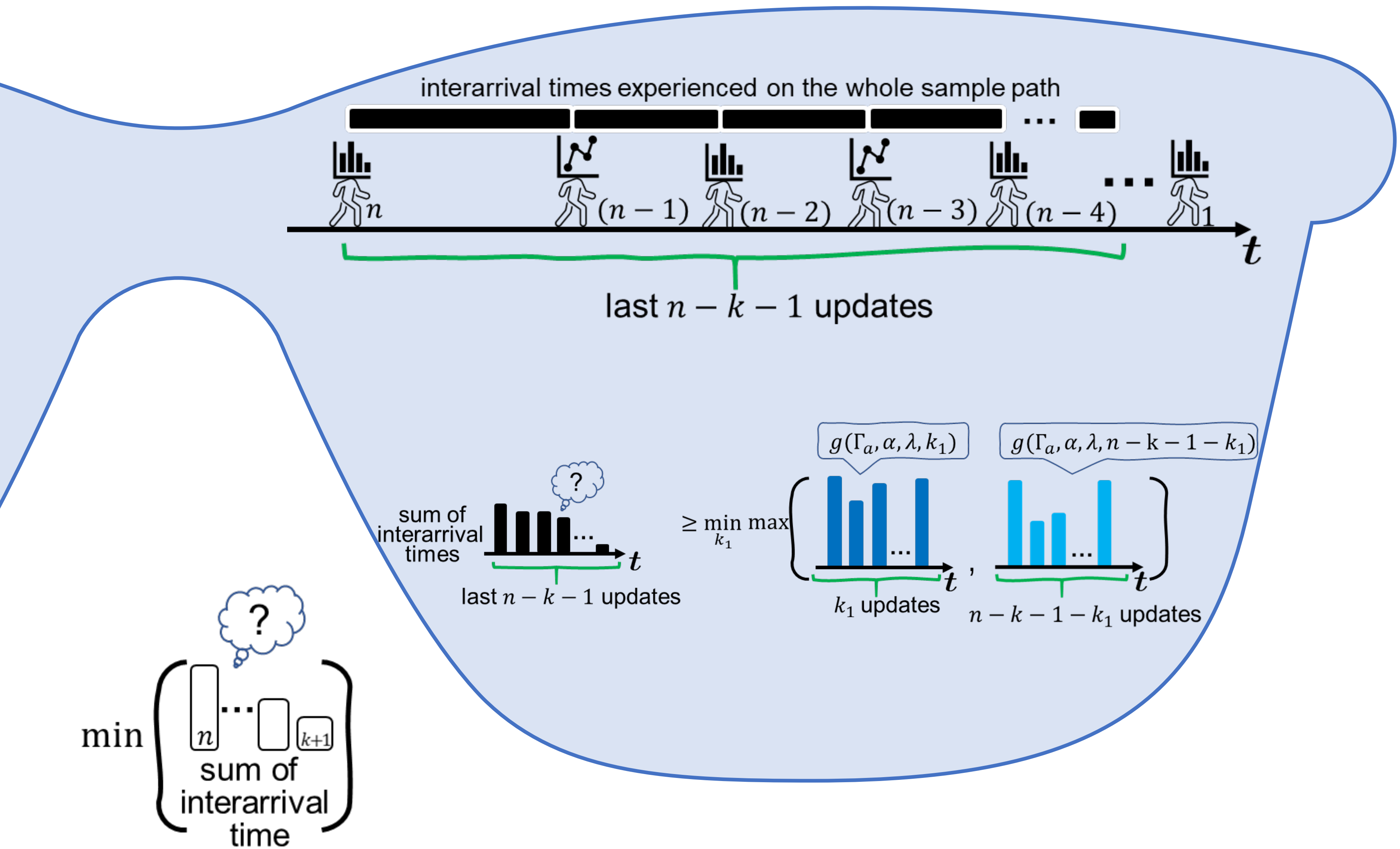
Our Solution



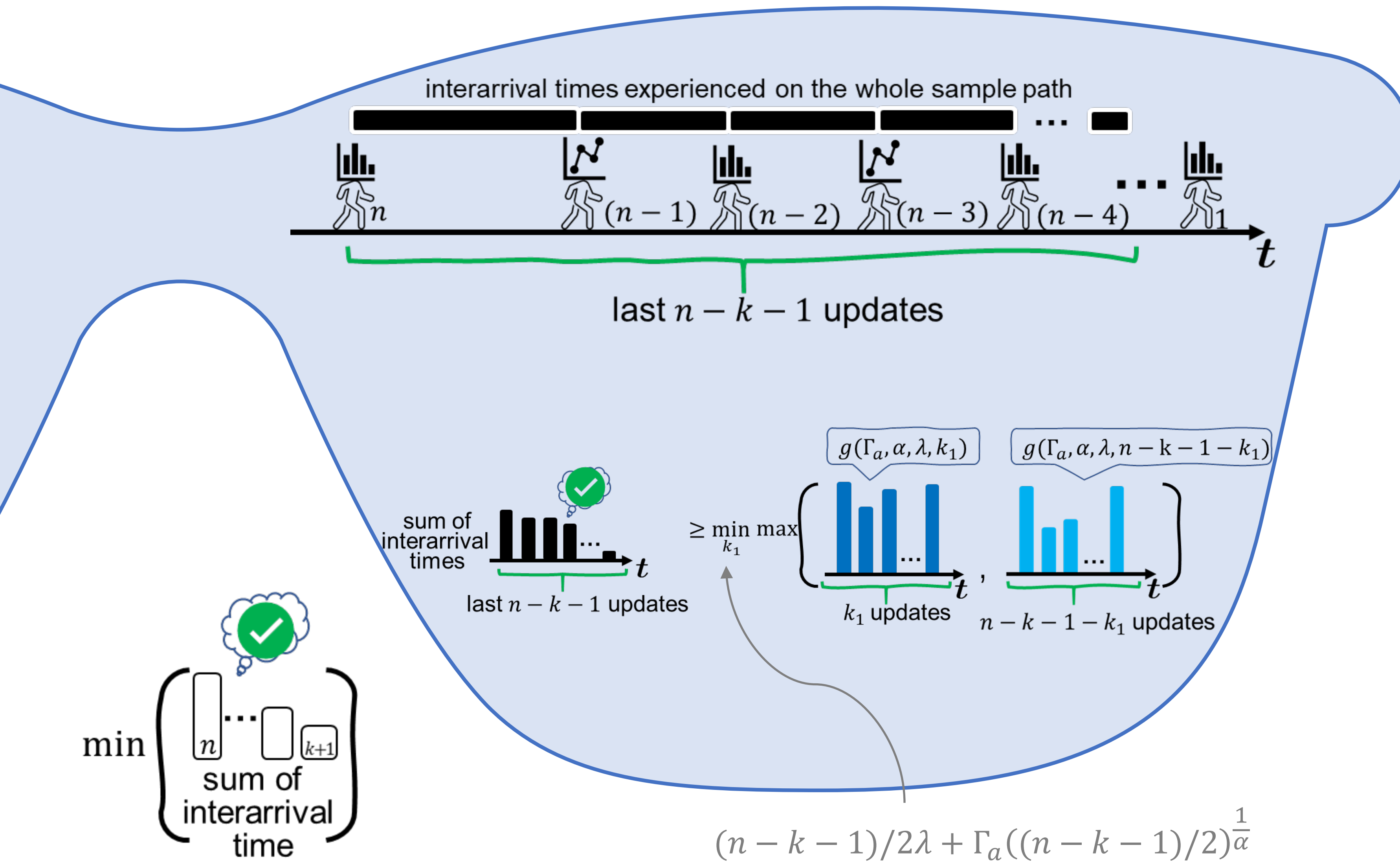
Our Solution



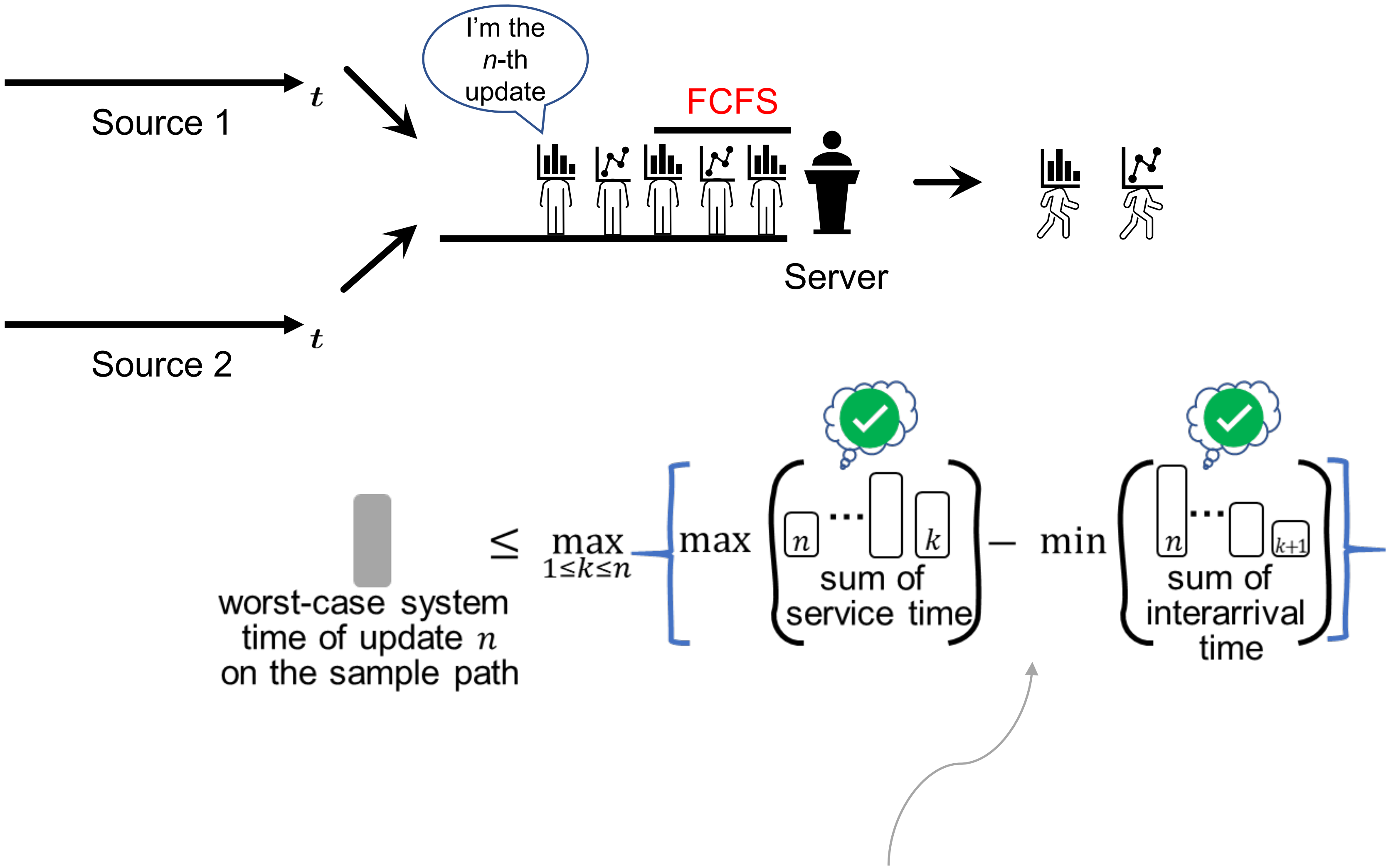
Our Solution



Our Solution

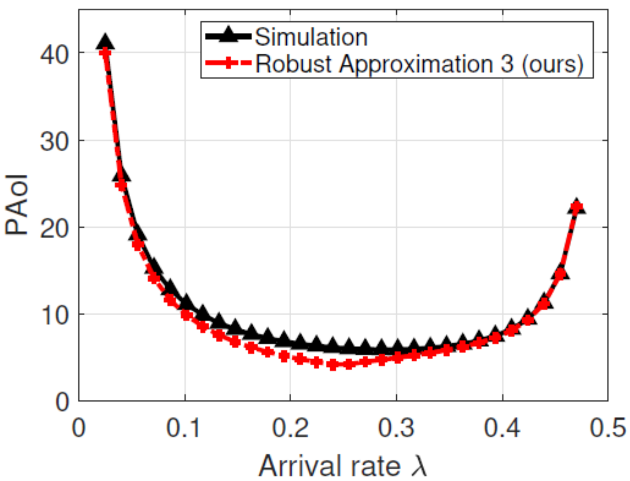


Worst-Case System Time

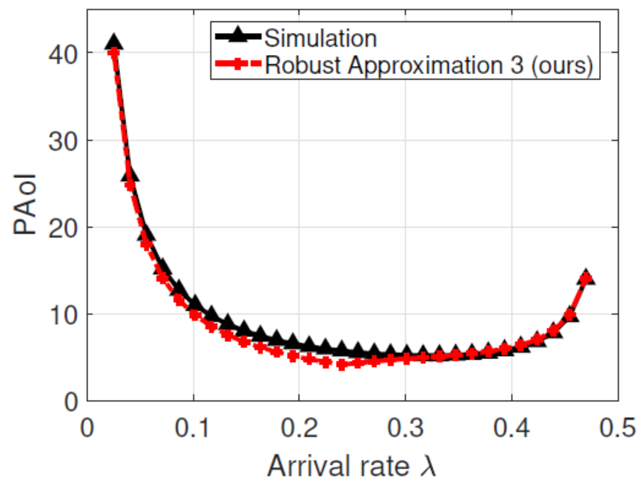


$$(n-k+1)/\mu - (n-k-1)/2\lambda + \Gamma_s(n-k+1)^{\frac{1}{\alpha}} + \Gamma_a((n-k-1)/2)^{\frac{1}{\alpha}}$$

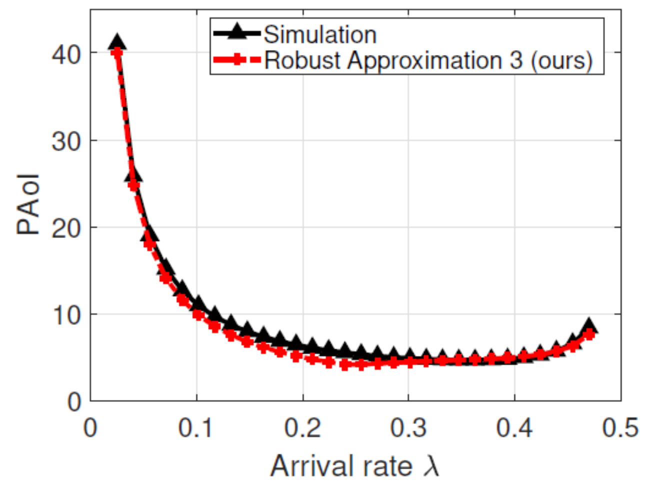
Numerical Result (Two-Source)



(a) Exponential distribution



(b) Normal distribution



(c) Uniform distribution

Methods	Exponential	Normal	Uniform
Robust Approx. 3 (ours)	**12.68%**	**10.05%**	**9.79%**

Error percentage

Conclusion

- Applied robust queueing theory to analyzing PAoI
  - Uncertainty sets
  - Worst-case analysis
- Single-source system
  - New robust bound of PAoI
- Two-source system
  - Resolve new challenges
  - Robust bound of PAoI

Conclusion

VIRGINIA TECH

- Applied robust queueing theory to analyzing PAoI
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Future Work

- Multiple-source system
- Asymmetric sources
- Heterogeneous tail coefficients (i.e., α)
- Dependence of arrival/service processes

Thank You!

Questions?

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